Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/167-6.2.3-e-x-^m-a+b-cosh-c+d-x^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [68]. This is test number [167].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (68)	0.00 (0)
Mathematica	100.00 (68)	0.00 (0)
Maxima	91.18 (62)	8.82 (6)
Fricas	88.24 (60)	11.76 (8)
Maple	85.29 (58)	14.71 (10)
Giac	63.24 (43)	36.76 (25)
Sympy	33.82 (23)	66.18 (45)
Mupad	30.88 (21)	69.12 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

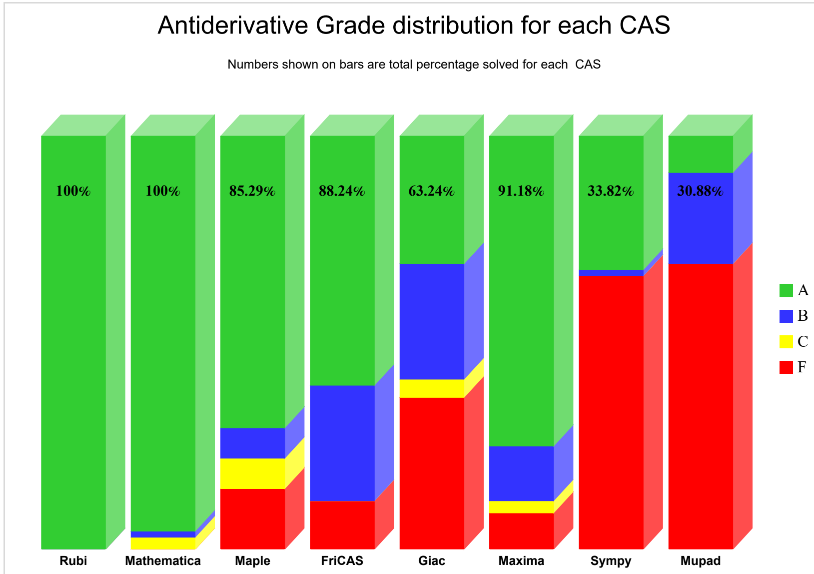
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

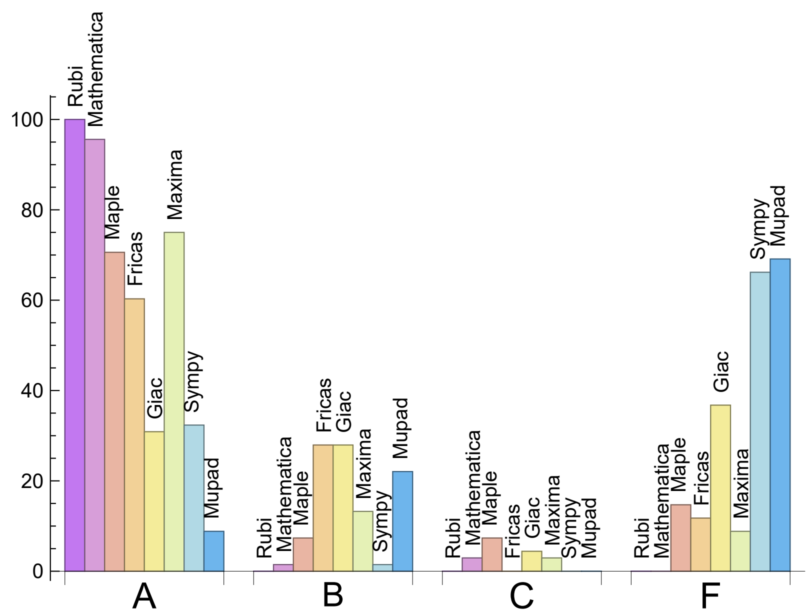
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	95.59	1.47	2.94	0.00
Maxima	75.00	13.24	2.94	8.82
Maple	70.59	7.35	7.35	14.71
Fricas	60.29	27.94	0.00	11.76
Sympy	32.35	1.47	0.00	66.18
Giac	30.88	27.94	4.41	36.76
Mupad	N/A	22.06	0.00	69.12

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	10	100.00 %	0.00 %	0.00 %
Fricas	8	100.00 %	0.00 %	0.00 %
Giac	25	100.00 %	0.00 %	0.00 %
Maxima	6	100.00 %	0.00 %	0.00 %
Sympy	45	93.33 %	4.44 %	2.22 %
Mupad	47	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

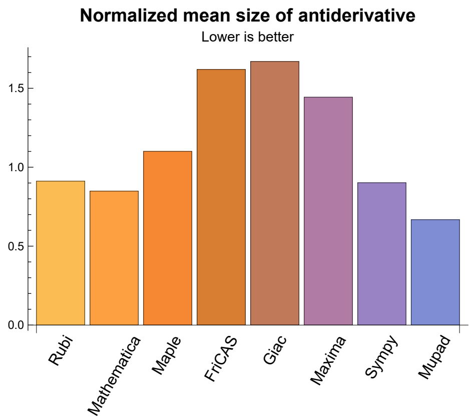
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

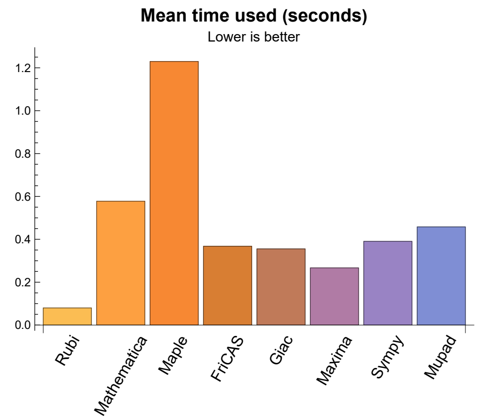
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	85.13	0.91	66.50	1.00
Mathematica	0.58	72.43	0.85	53.00	0.92
Maple	1.23	115.91	1.10	57.00	1.08
Maxima	0.27	111.19	1.44	61.00	0.95
Fricas	0.37	142.23	1.62	70.00	1.42
Sympy	0.39	49.74	0.90	29.00	1.00
Giac	0.36	162.21	1.67	65.00	1.70
Mupad	0.46	24.14	0.67	15.00	0.80

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{41, 42, 44, 46, 57, 58\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

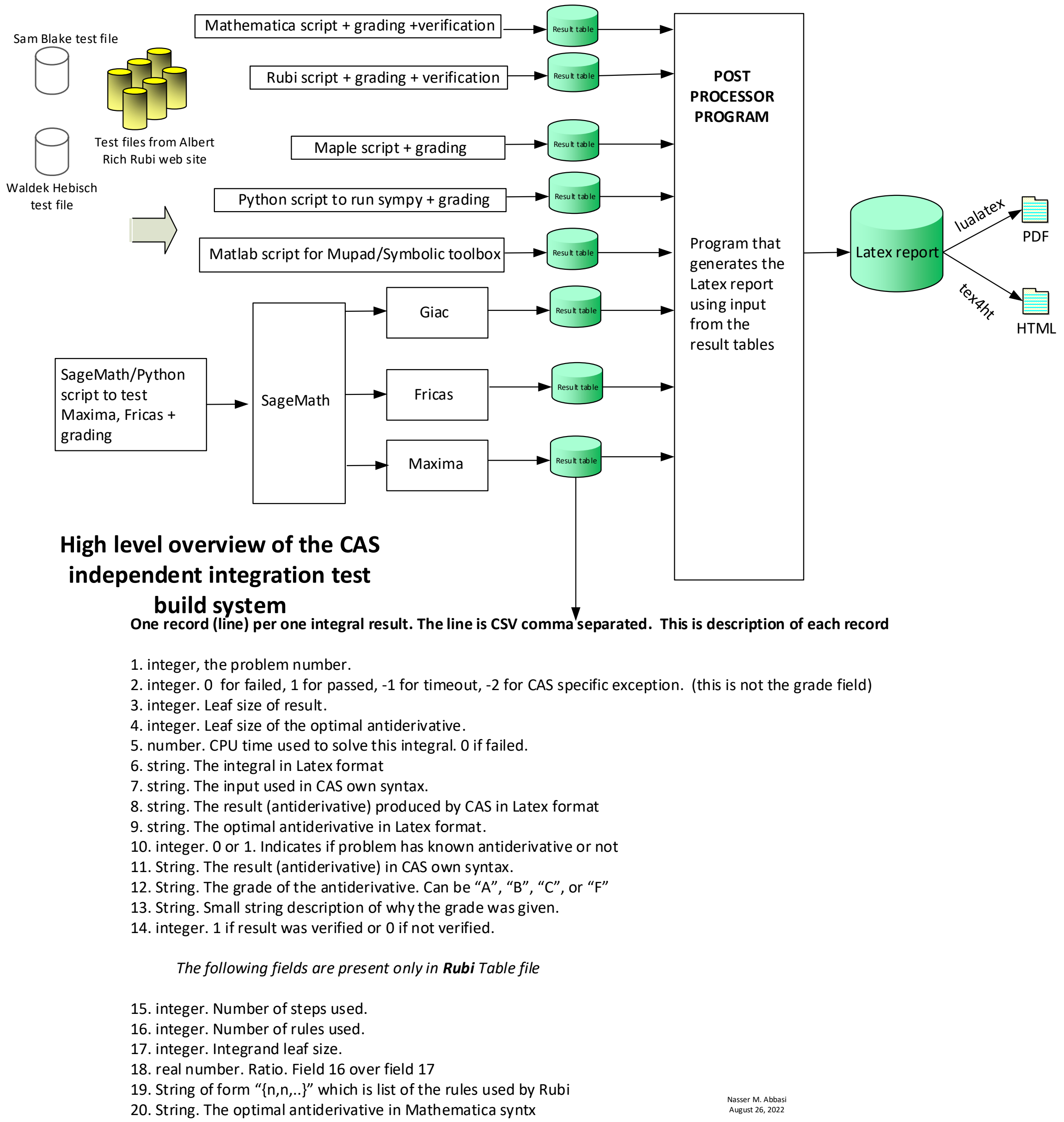
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66 }

B grade: { 3 }

C grade: { 67, 68 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 38, 40, 41, 42, 44, 46, 50, 51, 52, 53, 57, 58, 61, 66 }

B grade: { 29, 59, 60, 64, 65 }

C grade: { 35, 47, 54, 55, 56 }

F grade: { 37, 39, 43, 45, 48, 49, 62, 63, 67, 68 }

2.1.4 Maxima

A grade: { 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 46, 47, 48, 49, 50, 51, 52, 53, 57, 58, 59, 60, 64, 65, 66 }

B grade: { 1, 2, 4, 17, 22, 54, 55, 56, 61 }

C grade: { 28, 29 }

F grade: { 43, 45, 62, 63, 67, 68 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 5, 7, 8, 10, 11, 12, 14, 15, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 38, 40, 41, 42, 44, 46, 53, 54, 56, 57, 58, 59, 60, 61, 64, 65, 66 }

B grade: { 2, 6, 9, 13, 16, 20, 21, 22, 30, 34, 36, 50, 51, 52, 55, 62, 63, 67, 68 }

C grade: { }

F grade: { 35, 37, 39, 43, 45, 47, 48, 49 }

2.1.6 Sympy

A grade: { 1, 3, 8, 15, 17, 22, 23, 24, 26, 27, 28, 29, 33, 41, 42, 44, 57, 58, 59, 60, 61, 66 }

B grade: { 10 }

C grade: { }

F grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 25, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 67, 68 }

2.1.7 Giac

A grade: { 2, 4, 5, 9, 10, 11, 12, 16, 17, 18, 19, 22, 41, 42, 44, 46, 53, 57, 58, 61, 66 }

B grade: { 1, 3, 7, 8, 14, 15, 21, 23, 24, 25, 26, 27, 28, 29, 33, 59, 60, 64, 65 }

C grade: { 54, 55, 56 }

F grade: { 6, 13, 20, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 62, 63, 67, 68 }

2.1.8 Mupad

A grade: { 41, 42, 44, 46, 57, 58 }

B grade: { 1, 3, 8, 10, 15, 17, 22, 23, 24, 27, 28, 29, 33, 61, 66 }

C grade: { }

F grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 25, 26, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 62, 63, 64, 65, 67, 68 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	45	80	29	36	75	28
N.S.	1	1.00	0.91	1.32	2.35	0.85	1.06	2.21	0.82
time (sec)	N/A	0.024	0.030	0.359	0.268	0.350	0.170	0.412	0.083

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	67	74	110	189	0	75	-1
N.S.	1	1.00	0.97	1.07	1.59	2.74	0.00	1.09	-0.01
time (sec)	N/A	0.030	0.051	0.503	0.265	0.383	0.000	0.430	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	31	14	13	13	19	27	13
N.S.	1	1.00	2.07	0.93	0.87	0.87	1.27	1.80	0.87
time (sec)	N/A	0.012	0.009	0.404	0.263	0.380	0.077	0.383	0.048

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	45	40	87	49	0	41	-1
N.S.	1	1.00	0.85	0.75	1.64	0.92	0.00	0.77	-0.02
time (sec)	N/A	0.013	0.023	0.490	0.267	0.403	0.000	0.411	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	24	39	0	24	-1
N.S.	1	1.00	0.92	1.08	0.96	1.56	0.00	0.96	-0.04
time (sec)	N/A	0.024	0.012	0.425	0.322	0.374	0.000	0.419	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	70	55	183	0	0	-1
N.S.	1	1.00	1.06	1.06	0.83	2.77	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.051	0.506	0.256	0.399	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	58	40	71	0	108	-1
N.S.	1	1.00	0.90	1.38	0.95	1.69	0.00	2.57	-0.02
time (sec)	N/A	0.063	0.032	0.438	0.307	0.356	0.000	0.403	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	55	59	60	78	142	42
N.S.	1	1.00	0.78	1.08	1.16	1.18	1.53	2.78	0.82
time (sec)	N/A	0.037	0.078	1.531	0.264	0.353	0.283	0.402	0.105

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	101	90	95	427	0	97	-1
N.S.	1	1.00	1.02	0.91	0.96	4.31	0.00	0.98	-0.01
time (sec)	N/A	0.072	0.155	1.517	0.476	0.388	0.000	0.412	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	38	28	60	54	22
N.S.	1	1.00	0.87	1.10	1.23	0.90	1.94	1.74	0.71
time (sec)	N/A	0.020	0.020	0.651	0.268	0.401	0.132	0.391	0.058

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	86	51	56	73	0	58	-1
N.S.	1	1.00	1.10	0.65	0.72	0.94	0.00	0.74	-0.01
time (sec)	N/A	0.030	0.053	1.241	0.476	0.444	0.000	0.412	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	34	31	49	0	35	-1
N.S.	1	1.00	0.89	0.92	0.84	1.32	0.00	0.95	-0.03
time (sec)	N/A	0.042	0.016	1.231	0.303	0.358	0.000	0.415	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	94	86	61	394	0	0	-1
N.S.	1	1.00	1.07	0.98	0.69	4.48	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.177	1.364	0.320	0.415	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	69	36	90	0	126	-1
N.S.	1	1.00	0.81	1.21	0.63	1.58	0.00	2.21	-0.02
time (sec)	N/A	0.088	0.066	1.227	0.319	0.347	0.000	0.411	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	55	93	100	96	92	192	70
N.S.	1	1.00	0.70	1.18	1.27	1.22	1.16	2.43	0.89
time (sec)	N/A	0.053	0.121	1.861	0.264	0.397	0.425	0.411	0.957

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	184	157	162	903	0	166	-1
N.S.	1	1.00	1.15	0.98	1.01	5.64	0.00	1.04	-0.01
time (sec)	N/A	0.103	0.265	2.036	0.497	0.403	0.000	0.407	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	62	38	44	56	26
N.S.	1	1.00	1.00	0.94	1.88	1.15	1.33	1.70	0.79
time (sec)	N/A	0.022	0.009	1.072	0.256	0.364	0.192	0.423	0.062

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	136	86	91	113	0	95	-1
N.S.	1	1.00	1.09	0.69	0.73	0.90	0.00	0.76	-0.01
time (sec)	N/A	0.051	0.087	1.464	0.479	0.422	0.000	0.419	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	55	50	83	0	50	-1
N.S.	1	1.00	0.89	1.00	0.91	1.51	0.00	0.91	-0.02
time (sec)	N/A	0.067	0.026	1.629	0.332	0.338	0.000	0.413	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	204	149	102	891	0	0	-1
N.S.	1	1.00	1.50	1.10	0.75	6.55	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.257	1.819	0.336	0.445	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	92	120	58	168	0	224	-1
N.S.	1	1.00	1.01	1.32	0.64	1.85	0.00	2.46	-0.01
time (sec)	N/A	0.152	0.058	1.553	0.339	0.358	0.000	0.412	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	63	126	128	94	108	52
N.S.	1	1.00	1.00	0.94	1.88	1.91	1.40	1.61	0.78
time (sec)	N/A	0.036	0.024	1.553	0.260	0.407	0.953	0.420	0.988

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	15	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	1.88	0.75
time (sec)	N/A	0.011	0.006	0.464	0.271	0.380	0.123	0.417	0.879

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	8	15	15
N.S.	1	1.00	1.00	0.88	0.75	0.75	1.00	1.88	1.88
time (sec)	N/A	0.008	0.005	0.428	0.270	0.413	2.931	0.413	0.903

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	56	37	58	0	173	-1
N.S.	1	1.00	1.00	1.70	1.12	1.76	0.00	5.24	-0.03
time (sec)	N/A	0.054	0.020	1.334	0.302	0.362	0.000	0.417	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	27	24	39	17	43	-1
N.S.	1	1.00	1.00	1.29	1.14	1.86	0.81	2.05	-0.05
time (sec)	N/A	0.023	0.009	1.344	0.296	0.368	0.643	0.402	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	15	29	13
N.S.	1	1.00	1.00	1.08	1.00	1.15	1.15	2.23	1.00
time (sec)	N/A	0.011	0.005	0.407	0.257	0.566	0.327	0.408	0.875

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	44	47	33	29	93	29
N.S.	1	1.00	1.00	1.52	1.62	1.14	1.00	3.21	1.00
time (sec)	N/A	0.021	0.017	1.212	0.288	0.437	0.473	0.412	0.885

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	94	48	43	46	216	66
N.S.	1	1.00	0.85	2.04	1.04	0.93	1.00	4.70	1.43
time (sec)	N/A	0.038	0.031	1.217	0.296	0.395	0.693	0.411	0.931

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	71	70	72	225	0	0	-1
N.S.	1	1.00	1.06	1.04	1.07	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.059	0.866	0.289	0.396	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	24	39	0	0	-1
N.S.	1	1.00	1.00	1.08	0.96	1.56	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.012	0.785	0.291	0.462	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	44	63	53	0	0	-1
N.S.	1	1.00	0.86	0.77	1.11	0.93	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.026	0.865	0.303	0.422	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	22	33	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	1.47	2.20	0.87
time (sec)	N/A	0.012	0.006	0.407	0.270	0.447	0.661	0.407	0.901

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	82	63	250	0	0	-1
N.S.	1	1.00	0.99	1.09	0.84	3.33	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.052	0.876	0.306	0.514	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	77	74	61	0	0	0	-1
N.S.	1	1.00	1.15	1.10	0.91	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.054	0.449	0.073	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	33	30	54	0	0	-1
N.S.	1	1.00	0.92	1.32	1.20	2.16	0.00	0.00	-0.04
time (sec)	N/A	0.032	0.017	1.252	0.327	0.450	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	81	0	68	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.085	1.052	0.076	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	40	37	69	0	0	-1
N.S.	1	1.00	0.84	0.93	0.86	1.60	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.023	5.184	0.333	0.605	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	138	0	125	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	1.342	1.013	0.091	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	52	67	62	114	0	0	-1
N.S.	1	1.00	0.78	1.00	0.93	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.036	4.842	0.343	0.392	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	3.739	0.574	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	5.681	0.777	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.107	0.509	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.036	4.085	0.516	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	148	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.274	0.829	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	5.942	0.717	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	100	110	85	0	0	0	-1
N.S.	1	1.00	1.12	1.24	0.96	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.127	0.539	0.093	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	116	0	101	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.79	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.180	1.106	0.103	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	182	0	173	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.86	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	2.658	1.370	0.162	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	74	34	140	0	0	-1
N.S.	1	1.00	1.02	1.64	0.76	3.11	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.042	1.444	0.323	0.399	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	90	47	182	0	0	-1
N.S.	1	1.00	0.81	1.34	0.70	2.72	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.095	2.998	0.343	0.384	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	97	152	70	320	0	0	-1
N.S.	1	1.00	0.86	1.35	0.62	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.116	3.546	0.350	0.369	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	60	54	69	98	0	52	-1
N.S.	1	1.00	0.85	0.76	0.97	1.38	0.00	0.73	-0.01
time (sec)	N/A	0.034	0.528	0.476	0.316	0.365	0.000	0.408	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	62	136	818	164	0	137	-1
N.S.	1	1.00	0.55	1.20	7.24	1.45	0.00	1.21	-0.01
time (sec)	N/A	0.071	0.090	0.803	0.498	0.502	0.000	0.410	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	39	66	650	134	0	99	-1
N.S.	1	1.00	0.72	1.22	12.04	2.48	0.00	1.83	-0.02
time (sec)	N/A	0.040	0.024	0.770	0.462	0.443	0.000	0.425	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	36	478	54	0	39	-1
N.S.	1	1.00	0.68	0.97	12.92	1.46	0.00	1.05	-0.03
time (sec)	N/A	0.015	0.006	0.777	0.415	0.373	0.000	0.405	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	1.178	0.602	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.028	7.430	0.601	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	104	831	486	104	269	915	-1
N.S.	1	1.00	0.30	2.40	1.40	0.30	0.78	2.64	-0.00
time (sec)	N/A	0.304	0.375	1.662	0.273	0.497	0.317	0.422	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	72	303	291	68	151	300	-1
N.S.	1	1.00	0.43	1.81	1.74	0.41	0.90	1.80	-0.01
time (sec)	N/A	0.134	0.143	1.648	0.282	0.399	0.175	0.422	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	63	110	44	65	65	43
N.S.	1	1.00	0.93	1.17	2.04	0.81	1.20	1.20	0.80
time (sec)	N/A	0.034	0.048	1.388	0.262	0.434	0.158	0.403	0.957

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	127	0	0	217	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.391	0.632	0.000	0.353	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	199	0	0	317	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.278	1.291	0.604	0.000	0.468	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	537	537	381	1815	642	181	0	2163	-1
N.S.	1	1.00	0.71	3.38	1.20	0.34	0.00	4.03	-0.00
time (sec)	N/A	0.485	0.516	1.680	0.279	0.346	0.000	0.453	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	118	659	369	109	0	707	-1
N.S.	1	1.00	0.45	2.52	1.41	0.42	0.00	2.71	-0.00
time (sec)	N/A	0.222	0.292	1.657	0.286	0.420	0.000	0.428	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	133	136	58	94	128	75
N.S.	1	1.00	0.76	1.56	1.60	0.68	1.11	1.51	0.88
time (sec)	N/A	0.056	0.072	1.338	0.273	0.359	0.250	0.412	0.984

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	231	0	0	503	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	2.17	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.057	0.627	0.000	0.448	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	211	0	0	706	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.525	0.362	0.646	0.000	0.410	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [45] had the largest ratio of [22]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	12	0.250
2	A	4	4	1.00	12	0.333
3	A	2	2	1.00	10	0.200
4	A	3	3	1.00	8	0.375
5	A	3	3	1.00	12	0.250
6	A	4	4	1.00	12	0.333
7	A	5	5	1.00	12	0.417
8	A	3	3	1.00	14	0.214
9	A	6	5	1.00	14	0.357
10	A	3	3	1.00	12	0.250
11	A	5	4	1.00	10	0.400
12	A	5	4	1.00	14	0.286
13	A	6	6	1.00	14	0.429
14	A	7	6	1.00	14	0.429
15	A	4	4	1.00	14	0.286
16	A	10	5	1.00	14	0.357
17	A	3	2	1.00	12	0.167
18	A	8	4	1.00	10	0.400
19	A	8	4	1.00	14	0.286
20	A	9	5	1.00	14	0.357
21	A	12	6	1.00	14	0.429
22	A	3	2	1.00	12	0.167
23	A	2	2	1.00	8	0.250
24	A	2	2	1.00	8	0.250
25	A	5	5	1.00	8	0.625

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	3	1.00	12	0.250
27	A	2	2	1.00	12	0.167
28	A	3	3	1.00	12	0.250
29	A	4	3	1.00	12	0.250
30	A	5	5	1.00	8	0.625
31	A	3	3	1.00	12	0.250
32	A	4	4	1.00	12	0.333
33	A	2	2	1.00	12	0.167
34	A	5	5	1.00	12	0.417
35	A	3	2	1.00	8	0.250
36	A	3	3	1.00	12	0.250
37	A	5	3	1.00	10	0.300
38	A	5	4	1.00	14	0.286
39	A	8	3	1.00	10	0.300
40	A	8	4	1.00	14	0.286
41	A	0	0	0.00	0	0.000
42	A	0	0	0.00	0	0.000
43	A	3	3	1.00	20	0.150
44	A	0	0	0.00	0	0.000
45	A	5	5	1.00	22	0.227
46	A	0	0	0.00	0	0.000
47	A	3	2	1.00	12	0.167
48	A	5	3	1.00	14	0.214
49	A	8	3	1.00	14	0.214
50	A	5	5	1.00	16	0.312
51	A	7	6	1.00	18	0.333
52	A	12	6	1.00	18	0.333
53	A	4	4	1.00	18	0.222
54	A	12	9	1.00	12	0.750
55	A	8	7	1.00	10	0.700
56	A	4	4	1.00	8	0.500
57	A	0	0	0.00	0	0.000
58	A	0	0	0.00	0	0.000
59	A	16	4	1.00	18	0.222
60	A	10	4	1.00	16	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	4	1.00	14	0.286
62	A	10	5	1.00	18	0.278
63	A	11	6	1.00	18	0.333
64	A	23	6	1.00	18	0.333
65	A	13	5	1.00	16	0.312
66	A	5	4	1.00	14	0.286
67	A	13	5	1.00	18	0.278
68	A	14	6	1.00	18	0.333

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3 \cosh(a+bx^2)\,dx$

Optimal. Leaf size=34

$$-\frac{\cosh(a+bx^2)}{2b^2}+\frac{x^2\sinh(a+bx^2)}{2b}$$

```
[Out] -1/2*cosh(b*x^2+a)/b^2+1/2*x^2*sinh(b*x^2+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{5429, 3377, 2718\}$

$$\frac{x^2\sinh(a+bx^2)}{2b}-\frac{\cosh(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x^2],x]
```

```
[Out] -1/2*Cosh[a + b*x^2]/b^2 + (x^2*Sinh[a + b*x^2])/(2*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \cosh\left(a+bx^2\right) dx = \frac{1}{2}\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, x^2\right)$$
$$= \frac{x^2\sinh\left(a+bx^2\right)}{2b} - \frac{\text{Subst}\left(\int \sinh(a+bx)\,dx, x, x^2\right)}{2b}$$
$$= -\frac{\cosh\left(a+bx^2\right)}{2b^2} + \frac{x^2\sinh\left(a+bx^2\right)}{2b}$$

Mathematica [**A**]
time = 0.03, size = 31, normalized size = 0.91

$$\frac{-\cosh\left(a+bx^2\right)+bx^2\sinh\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x^2],x]
```

```
[Out] (-Cosh[a + b*x^2] + b*x^2*Sinh[a + b*x^2])/(2*b^2)
```

Maple [**A**]
time = 0.36, size = 45, normalized size = 1.32

method	result	size
risch	$\frac{(x^2b-1)e^{x^2b+a}}{4b^2}-\frac{(x^2b+1)e^{-x^2b-a}}{4b^2}$	45
meijerg	$-\frac{\cosh(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh\left(x^2b\right)}{2\sqrt{\pi}}-\frac{x^2b\sinh\left(x^2b\right)}{2\sqrt{\pi}}\right)}{b^2}+\frac{\sinh(a)(\cosh(x^2b)x^2b-\sinh(x^2b))}{2b^2}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2-1)/b^2*exp(b*x^2+a)-1/4*(b*x^2+1)/b^2*exp(-b*x^2-a)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(30) = 60.
time = 0.27, size = 80, normalized size = 2.35

$$\frac{1}{4}x^4\cosh\left(bx^2+a\right)-\frac{1}{8}b\left(\frac{(b^2x^4e^a-2bx^2e^a+2e^a)e^{(bx^2)}}{b^3}+\frac{(b^2x^4+2bx^2+2)e^{(-bx^2-a)}}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*cosh(b*x^2 + a) - 1/8*b*((b^2*x^4*e^a - 2*b*x^2*e^a + 2*e^a)*e^(b*x
^2)/b^3 + (b^2*x^4 + 2*b*x^2 + 2)*e^(-b*x^2 - a)/b^3)
```

Fricas [A]
time = 0.35, size = 29, normalized size = 0.85

$$\frac{bx^2 \sinh\left(bx^2+a\right) - \cosh\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*sinh(b*x^2 + a) - cosh(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.17, size = 36, normalized size = 1.06

$$\begin{cases} \frac{x^2 \sinh\left(a+bx^2\right)}{2b} - \frac{\cosh\left(a+bx^2\right)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^4 \cosh\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x**2+a),x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x**2)/(2*b) - cosh(a + b*x**2)/(2*b**2), Ne(b, 0
)), (x**4*cosh(a)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(30) = 60.
time = 0.41, size = 75, normalized size = 2.21

$$\frac{(bx^2+a-1)e^{(bx^2+a)} - (bx^2+a+1)e^{(-bx^2-a)}}{4\,b^2} - \frac{ae^{(bx^2+a)} - ae^{(-bx^2-a)}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 + a - 1)*e^(b*x^2 + a) - (b*x^2 + a + 1)*e^(-b*x^2 - a))/b^2 -
1/4*(a*e^(b*x^2 + a) - a*e^(-b*x^2 - a))/b^2
```

Mupad [B]
time = 0.08, size = 28, normalized size = 0.82

$$-\frac{\cosh(b\,x^2+a) - b\,x^2 \sinh(b\,x^2+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x^2),x)
```

```
[Out] -(cosh(a + b*x^2) - b*x^2*sinh(a + b*x^2))/(2*b^2)
```

3.2 $\int x^2 \cosh\left(a+bx^2\right)\,dx$

Optimal. Leaf size=69

$$\frac{e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)}{8b^{3/2}}-\frac{e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{8b^{3/2}}+\frac{x\sinh\left(a+bx^2\right)}{2b}$$

```
[Out] 1/2*x*sinh(b*x^2+a)/b+1/8*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)-1/8*exp(a)
*erfi(x*b^(1/2))*Pi^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5433, 5406, 2235, 2236}

$$\frac{\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\sqrt{b}\,x\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{8b^{3/2}}+\frac{x\sinh\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*b^(3/2)*E^a) - (E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/
(8*b^(3/2)) + (x*Sinh[a + b*x^2])/(2*b)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
```

```
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rubi steps

$$\begin{aligned}\int x^2\cosh\left(a+bx^2\right)\,dx&=\frac{x\sinh\left(a+bx^2\right)}{2b}-\frac{\int\sinh\left(a+bx^2\right)\,dx}{2b}\\&=\frac{x\sinh\left(a+bx^2\right)}{2b}+\frac{\int e^{-a-bx^2}\,dx}{4b}-\frac{\int e^{a+bx^2}\,dx}{4b}\\&=\frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)}{8b^{3/2}}-\frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}\,x\right)}{8b^{3/2}}+\frac{x\sinh\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 67, normalized size = 0.97

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)(\cosh(a)-\sinh(a))-\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)(\cosh(a)+\sinh(a))+4\sqrt{b}\,x\sinh\left(a+bx^2\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) - Sqrt[Pi]*Erfi[Sqrt[b]*x]*(Co
sh[a] + Sinh[a]) + 4*Sqrt[b]*x*Sinh[a + b*x^2])/(8*b^(3/2))
```

Maple [**A**]
time = 0.50, size = 74, normalized size = 1.07

method	result
risch	$-\frac{\mathrm{e}^{-a}x\,\mathrm{e}^{-x^2b}}{4b}+\frac{\mathrm{e}^{-a}\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{b}\right)}{8b^{\frac{3}{2}}}+\frac{\mathrm{e}^{a}\mathrm{e}^{x^2b}x}{4b}-\frac{\mathrm{e}^{a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}\,x\right)}{8b\sqrt{-b}}$
meijerg	$-\frac{i\cosh(a)\sqrt{\pi}\,\sqrt{2}\left(\frac{x\sqrt{2}\,(ib)^{\frac{3}{2}}\mathrm{e}^{x^2b}}{4\sqrt{\pi}\,b}-\frac{x\sqrt{2}\,(ib)^{\frac{3}{2}}\mathrm{e}^{-x^2b}}{4\sqrt{\pi}\,b}+\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\mathrm{erf}\left(x\sqrt{b}\right)}{8b^{\frac{3}{2}}}-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\mathrm{erfi}\left(x\sqrt{b}\right)}{8b^{\frac{3}{2}}}\right)}{2b\sqrt{ib}}-\frac{\sinh(a)\sqrt{\pi}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*exp(-a)/b*x*exp(-x^2*b)+1/8*exp(-a)/b^(3/2)*Pi^(1/2)*erf(x*b^(1/2))+1/
4*exp(a)*exp(x^2*b)*x/b-1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(49) = 98$.
time = 0.27, size = 110, normalized size = 1.59

$$\frac{1}{3}\,x^{3}\cosh\left(bx^{2}+a\right)-\frac{1}{24}\,b\left(\frac{2\,(2\,bx^{3}e^{a}-3\,xe^{a})e^{(bx^{2})}}{b^{2}}+\frac{2\,(2\,bx^{3}+3\,x)e^{(-bx^{2}-a)}}{b^{2}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,x\right)e^{(-a)}}{b^{\frac{5}{2}}}+\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)e^{a}}{\sqrt{-b}\,b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*cosh(b*x^2 + a) - 1/24*b*(2*(2*b*x^3*e^a - 3*x*e^a)*e^(b*x^2)/b^2 +
 2*(2*b*x^3 + 3*x)*e^(-b*x^2 - a)/b^2 - 3*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^
(5/2) + 3*sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b^2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(49) = 98$.
time = 0.38, size = 189, normalized size = 2.74

$$\frac{2\,bx\cosh\left(bx^{2}+a\right)^{2}+4\,bx\cosh\left(bx^{2}+a\right)\sinh\left(bx^{2}+a\right)+2\,bx\sinh\left(bx^{2}+a\right)^{2}+\sqrt{\pi}\,(\cosh\left(bx^{2}+a\right)\cosh\left(a\right)+(\cosh\left(a\right)+\sinh\left(a\right))\sinh\left(bx^{2}+a\right)+\cosh\left(bx^{2}+a\right)\sinh\left(a\right))\sqrt{-b}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)+\sqrt{\pi}\,(\cosh\left(bx^{2}+a\right)\cosh\left(a\right)+(\cosh\left(a\right)-\sinh\left(a\right))\sinh\left(bx^{2}+a\right)-\cosh\left(bx^{2}+a\right)\sinh\left(a\right))\sqrt{b}\,\operatorname{erf}\left(\sqrt{b}\,x\right)-2\,bx}{8\,(b^{2}\cosh\left(bx^{2}+a\right)+b^{2}\sinh\left(bx^{2}+a\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*x*cosh(b*x^2 + a)^2 + 4*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a) + 2*b*
x*sinh(b*x^2 + a)^2 + sqrt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) + sinh(a
))*sinh(b*x^2 + a) + cosh(b*x^2 + a)*sinh(a))*sqrt(-b)*erf(sqrt(-b)*x) + sq
rt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a) - cos
h(b*x^2 + a)*sinh(a))*sqrt(b)*erf(sqrt(b)*x) - 2*b*x)/(b^2*cosh(b*x^2 + a)
+ b^2*sinh(b*x^2 + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\cosh\left(a+bx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x**2+a),x)
```

```
[Out] Integral(x**2*cosh(a + b*x**2), x)
```

Giac [A]
time = 0.43, size = 75, normalized size = 1.09

$$\frac{xe^{(bx^{2}+a)}}{4\,b}-\frac{xe^{(-bx^{2}-a)}}{4\,b}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{b}\,x\right)e^{(-a)}}{8\,b^{\frac{3}{2}}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-b}\,x\right)e^{a}}{8\,\sqrt{-b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*x*e^(b*x^2 + a)/b - 1/4*x*e^(-b*x^2 - a)/b - 1/8*sqrt(pi)*erf(-sqrt(b)*
x)*e^(-a)/b^(3/2) + 1/8*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cosh\left(b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x^2),x)
```

```
[Out] int(x^2*cosh(a + b*x^2), x)
```

3.3 $\int x\cosh\left(a+bx^2\right)\,dx$

Optimal. Leaf size=15

$$\frac{\sinh\left(a+bx^2\right)}{2b}$$

```
[Out] 1/2*sinh(b*x^2+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5429, 2717}

$$\frac{\sinh\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x^2],x]
```

```
[Out] Sinh[a + b*x^2]/(2*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\cosh\left(a+bx^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \cosh(a+bx)\,dx, x, x^2\right)\\ &= \frac{\sinh\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.
time = 0.01, size = 31, normalized size = 2.07

$$\frac{\cosh\left(bx^2\right)\sinh(a)}{2b}+\frac{\cosh(a)\sinh\left(bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x^2],x]
[Out] (Cosh[b*x^2]*Sinh[a])/(2*b) + (Cosh[a]*Sinh[b*x^2])/(2*b)
```

Maple [A]
time = 0.40, size = 14, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\sinh(x^2b+a)}{2b}$	14
default	$\frac{\sinh(x^2b+a)}{2b}$	14
risch	$\frac{e^{x^2b+a}}{4b}-\frac{e^{-x^2b-a}}{4b}$	31
meijerg	$\frac{\cosh(a)\sinh(x^2b)}{2b}-\frac{\sinh(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(x^2b)}{\sqrt{\pi}}\right)}{2b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*sinh(b*x^2+a)/b
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.87

$$\frac{\sinh(bx^2+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*sinh(b*x^2 + a)/b
```

Fricas [A]
time = 0.38, size = 13, normalized size = 0.87

$$\frac{\sinh(bx^2+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*sinh(b*x^2 + a)/b
```

Sympy [A]
time = 0.08, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\sinh\left(a+bx^2\right)}{2b} & \text{for } b \neq 0 \\ \frac{x^2\cosh\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x**2+a),x)
```

```
[Out] Piecewise((sinh(a + b*x**2)/(2*b), Ne(b, 0)), (x**2*cosh(a)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.
time = 0.38, size = 27, normalized size = 1.80

$$\frac{e^{\left(bx^2+a\right)} - e^{\left(-bx^2-a\right)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(e^(b*x^2 + a) - e^(-b*x^2 - a))/b
```

Mupad [B]
time = 0.05, size = 13, normalized size = 0.87

$$\frac{\sinh(b\,x^2 + a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x^2),x)
```

```
[Out] sinh(a + b*x^2)/(2*b)
```

3.4 $\int \cosh(a+bx^2)\,dx$

Optimal. Leaf size=53

$$\frac{e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}$$

```
[Out] 1/4*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/4*exp(a)*erfi(x*b^(1/2))*Pi^(1
/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5407, 2235, 2236}

$$\frac{\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}+\frac{\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[b]*x])/(4*Sqrt[b]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/
(4*Sqrt[b])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rubi steps

$$\int \cosh\left(a+bx^2\right)\,dx = \frac{1}{2}\int e^{-a-bx^2}\,dx + \frac{1}{2}\int e^{a+bx^2}\,dx$$
$$= \frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)}{4\sqrt{b}} + \frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.85

$$\frac{\sqrt{\pi}\left(\mathrm{Erf}\left(\sqrt{b}\,x\right)(\cosh(a)-\sinh(a))+\mathrm{Erfi}\left(\sqrt{b}\,x\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + Erfi[Sqrt[b]*x]*(Cosh[a] +
Sinh[a])))/(4*Sqrt[b])
```

Maple [A]
time = 0.49, size = 40, normalized size = 0.75

method	result
risch	$\frac{\mathrm{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\,\mathrm{e}^{-a}}{4\sqrt{b}}+\frac{\mathrm{e}^{a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}\,x\right)}{4\sqrt{-b}}$
meijerg	$\frac{\cosh(a)\sqrt{\pi}\,\sqrt{2}\left(\frac{\sqrt{ib}\,\sqrt{2}\,\mathrm{erf}\left(x\sqrt{b}\right)}{2\sqrt{b}}+\frac{\sqrt{ib}\,\sqrt{2}\,\mathrm{erfi}\left(x\sqrt{b}\right)}{2\sqrt{b}}\right)}{4\sqrt{ib}}-\frac{i\sinh(a)\sqrt{\pi}\,\sqrt{2}\left(-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\mathrm{erf}\left(x\sqrt{b}\right)}{2b^{\frac{3}{2}}}\right.}{4\sqrt{ib}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*erf(x*b^(1/2))*Pi^(1/2)*exp(-a)/b^(1/2)+1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)*
erf((-b)^(1/2)*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(35) = 70.
time = 0.27, size = 87, normalized size = 1.64

$$-\frac{1}{4}\,b\left(\frac{2\,xe^{\left(bx^2+a\right)}}{b}+\frac{2\,xe^{\left(-bx^2-a\right)}}{b}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)e^{(-a)}}{b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}\,x\right)e^{a}}{\sqrt{-b}\,b}\right)+x\cosh\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*x*e^(b*x^2 + a)/b + 2*x*e^(-b*x^2 - a)/b - sqrt(pi)*erf(sqrt(b)*x
)*e^(-a)/b^(3/2) - sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)) + x*cosh(b*x^
2 + a)
```

Fricas [A]
time = 0.40, size = 49, normalized size = 0.92

$$-\frac{\sqrt{\pi}\,\sqrt{-b}\,(\cosh(a)+\sinh(a))\,\mathrm{erf}\left(\sqrt{-b}\,x\right)-\sqrt{\pi}\,\sqrt{b}\,(\cosh(a)-\sinh(a))\,\mathrm{erf}\left(\sqrt{b}\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) - sqrt(pi)*sqrt
(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(a+bx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a),x)
```

```
[Out] Integral(cosh(a + b*x**2), x)
```

Giac [A]
time = 0.41, size = 41, normalized size = 0.77

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}\,x\right)e^{(-a)}}{4\,\sqrt{b}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}\,x\right)e^{a}}{4\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/sqrt(b) - 1/4*sqrt(pi)*erf(-sqrt(-b)*x
)*e^a/sqrt(-b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\cosh\left(b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2),x)
[Out] int(cosh(a + b*x^2), x)
```

3.5 $\int \frac{\cosh(a+bx^2)}{x} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\cosh(a)\text{Chi}\left(bx^2\right)+\frac{1}{2}\sinh(a)\text{Shi}\left(bx^2\right)$$

```
[Out] 1/2*Chi(b*x^2)*cosh(a)+1/2*Shi(b*x^2)*sinh(a)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5427, 5425, 5424}

$$\frac{1}{2}\cosh(a)\text{Chi}\left(bx^2\right)+\frac{1}{2}\sinh(a)\text{Shi}\left(bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]/x,x]
```

```
[Out] (Cosh[a]*CoshIntegral[b*x^2])/2 + (Sinh[a]*SinhIntegral[b*x^2])/2
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh\left(a+bx^2\right)}{x}\,dx &= \cosh(a)\int \frac{\cosh\left(bx^2\right)}{x}\,dx+\sinh(a)\int \frac{\sinh\left(bx^2\right)}{x}\,dx\\
&= \frac{1}{2}\cosh(a)\text{Chi}\left(bx^2\right)+\frac{1}{2}\sinh(a)\text{Shi}\left(bx^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.92

$$\frac{1}{2}\left(\cosh(a)\text{Chi}\left(bx^2\right)+\sinh(a)\text{Shi}\left(bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]/x,x]
[Out] (Cosh[a]*CoshIntegral[b*x^2] + Sinh[a]*SinhIntegral[b*x^2])/2
```

Maple [A]
time = 0.42, size = 27, normalized size = 1.08

method	result	size
risch	$-\frac{e^{-a}\,\text{expIntegral}\left(1,x^2 b\right)}{4}-\frac{e^{a}\,\text{expIntegral}\left(1,-x^2 b\right)}{4}$	27
meijerg	$\frac{\cosh(a)\sqrt{\pi}\left(\frac{2\,\text{hyperbolicCosineIntegral}\left(x^2 b\right)-2\ln\left(x^2 b\right)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+4\ln(x)+2\ln(ib)}{\sqrt{\pi}}\right)}{4}+\frac{\text{hyperbolicSineIntegral}\left(x^2 b\right)\sinh(a)}{2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)/x,x,method=_RETURNVERBOSE)
[Out] -1/4*exp(-a)*Ei(1,x^2*b)-1/4*exp(a)*Ei(1,-x^2*b)
```

Maxima [A]
time = 0.32, size = 24, normalized size = 0.96

$$\frac{1}{4}\,\text{Ei}\left(-bx^2\right)e^{(-a)}+\frac{1}{4}\,\text{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)/x,x, algorithm="maxima")
[Out] 1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

Fricas [A]
time = 0.37, size = 39, normalized size = 1.56

$$\frac{1}{4}\left(\text{Ei}\left(bx^2\right)+\text{Ei}\left(-bx^2\right)\right)\cosh\left(a\right)+\frac{1}{4}\left(\text{Ei}\left(bx^2\right)-\text{Ei}\left(-bx^2\right)\right)\sinh\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)/x,x, algorithm="fricas")
[Out] 1/4*(Ei(b*x^2) + Ei(-b*x^2))*cosh(a) + 1/4*(Ei(b*x^2) - Ei(-b*x^2))*sinh(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)/x,x)
```

```
[Out] Integral(cosh(a + b*x**2)/x, x)
```

Giac [A]
time = 0.42, size = 24, normalized size = 0.96

$$\frac{1}{4}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+\frac{1}{4}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\frac{\cosh(a)\ \mathrm{coshint}(b\,x^2)}{2}+\frac{\sinh(a)\ \mathrm{sinhint}(b\,x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)/x,x)
```

```
[Out] (cosh(a)*coshint(b*x^2))/2 + (sinh(a)*sinhint(b*x^2))/2
```

3.6 $\int \frac{\cosh(a+bx^2)}{x^2}\,dx$

Optimal. Leaf size=66

$$-\frac{\cosh\left(a+bx^2\right)}{x}-\frac{1}{2}\sqrt{b}\,e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{b}\,e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)$$

```
[Out] -cosh(b*x^2+a)/x-1/2*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)+1/2*exp(a)*erfi
(x*b^(1/2))*b^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5435, 5406, 2235, 2236}

$$-\frac{1}{2}\sqrt{\pi}\,e^{-a}\sqrt{b}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{\pi}\,e^{a}\sqrt{b}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)-\frac{\cosh\left(a+bx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]/x^2,x]
```

```
[Out] -(Cosh[a + b*x^2]/x) - (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(2*E^a) + (Sqrt[b]
*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/2
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5435

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x
)^(m + 1)*(Cosh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
```

```
&& LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx^2)}{x^2}\,dx &= -\frac{\cosh(a+bx^2)}{x}+(2b)\int \sinh(a+bx^2)\,dx\\ &= -\frac{\cosh(a+bx^2)}{x}-b\int e^{-a-bx^2}\,dx+b\int e^{a+bx^2}\,dx\\ &= -\frac{\cosh(a+bx^2)}{x}-\frac{1}{2}\sqrt{b}\,e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{b}\,e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}\,x\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 70, normalized size = 1.06

$$\frac{-2\cosh(a+bx^2)+\sqrt{b}\,\sqrt{\pi}\,x\mathrm{Erf}\left(\sqrt{b}\,x\right)(-\cosh(a)+\sinh(a))+\sqrt{b}\,\sqrt{\pi}\,x\mathrm{Erfi}\left(\sqrt{b}\,x\right)(\cosh(a)+\sinh(a))}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]/x^2,x]
```

```
[Out] (-2*Cosh[a + b*x^2] + Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a]
) + Sqrt[b]*Sqrt[Pi]*x*Erfi[Sqrt[b]*x]*(Cosh[a] + Sinh[a]))/(2*x)
```

Maple [A]
time = 0.51, size = 70, normalized size = 1.06

method	result
risch	$-\frac{e^{-a}e^{-x^2 b}}{2x}-\frac{e^{-a}\sqrt{b}\,\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{b}\right)}{2}-\frac{e^{a}e^{x^2 b}}{2x}+\frac{e^{a}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}\,x\right)}{2\sqrt{-b}}$
meijerg	$\frac{i\cosh(a)\sqrt{\pi}\,b\sqrt{2}\left(-\frac{2\sqrt{2}\,e^{x^2 b}}{\sqrt{\pi}\,x\sqrt{ib}}-\frac{2\sqrt{2}\,e^{-x^2 b}}{\sqrt{\pi}\,x\sqrt{ib}}-\frac{2\sqrt{2}\,\sqrt{b}\,\mathrm{erf}\left(x\sqrt{b}\right)}{\sqrt{ib}}+\frac{2\sqrt{2}\,\sqrt{b}\,\mathrm{erfi}\left(x\sqrt{b}\right)}{\sqrt{ib}}\right)}{8\sqrt{ib}}+\frac{\sinh(a)\sqrt{\pi}\,b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-a)/x*exp(-x^2*b)-1/2*exp(-a)*b^(1/2)*Pi^(1/2)*erf(x*b^(1/2))-1/2*
exp(a)*exp(x^2*b)/x+1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Maxima [A]
time = 0.26, size = 55, normalized size = 0.83

$$-\frac{1}{2}\left(\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,x\right)e^{(-a)}}{\sqrt{b}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)e^{a}}{\sqrt{-b}}\right)b-\frac{\cosh\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/sqrt(b) - sqrt(pi)*erf(sqrt(-b)*x)*e^a
/sqrt(-b))*b - cosh(b*x^2 + a)/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(48) = 96$.
time = 0.40, size = 183, normalized size = 2.77

$$-\frac{\sqrt{\pi}\left(x\cosh\left(bx^2+a\right)\cosh\left(a\right)+x\cosh\left(bx^2+a\right)\sinh\left(a\right)+\left(x\cosh\left(a\right)+x\sinh\left(a\right)\right)\sinh\left(bx^2+a\right)\right)\sqrt{-b}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)+\sqrt{\pi}\left(x\cosh\left(bx^2+a\right)\cosh\left(a\right)-x\cosh\left(bx^2+a\right)\sinh\left(a\right)+\left(x\cosh\left(a\right)-x\sinh\left(a\right)\right)\sinh\left(bx^2+a\right)\right)\sqrt{b}\,\operatorname{erf}\left(\sqrt{b}\,x\right)+\cosh\left(bx^2+a\right)^2+2\,\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)+\sinh\left(bx^2+a\right)^2+1}{2\left(x\cosh\left(bx^2+a\right)+x\sinh\left(bx^2+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2 + a)*sinh(a) + (x*
cosh(a) + x*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + sqrt(pi)*(
x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*x^2 + a)*sinh(a) + (x*cosh(a) - x*sinh
(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + cosh(b*x^2 + a)^2 + 2*cosh(b
*x^2 + a)*sinh(b*x^2 + a) + sinh(b*x^2 + a)^2 + 1)/(x*cosh(b*x^2 + a) + x*s
inh(b*x^2 + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)/x**2,x)
```

```
[Out] Integral(cosh(a + b*x**2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x^2 + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(b\,x^2+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)/x^2,x)
```

```
[Out] int(cosh(a + b*x^2)/x^2, x)
```

3.7 $\int \frac{\cosh(a+bx^2)}{x^3} \, dx$

Optimal. Leaf size=42

$$-\frac{\cosh(a+bx^2)}{2x^2}+\frac{1}{2}b\text{Chi}(bx^2)\sinh(a)+\frac{1}{2}b\cosh(a)\text{Shi}(bx^2)$$

```
[Out] -1/2*cosh(b*x^2+a)/x^2+1/2*b*cosh(a)*Shi(b*x^2)+1/2*b*Chi(b*x^2)*sinh(a)
```

Rubi [A]
time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5429, 3378, 3384, 3379, 3382}

$$\frac{1}{2}b\sinh(a)\text{Chi}(bx^2)+\frac{1}{2}b\cosh(a)\text{Shi}(bx^2)-\frac{\cosh(a+bx^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]/x^3,x]
```

```
[Out] -1/2*Cosh[a + b*x^2]/x^2 + (b*CoshIntegral[b*x^2]*Sinh[a])/2 + (b*Cosh[a]*S
inhIntegral[b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(a+bx^2)}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\cosh(a+bx^2)}{2x^2} + \frac{1}{2}b\text{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx, x, x^2\right)\\
&= -\frac{\cosh(a+bx^2)}{2x^2} + \frac{1}{2}(b\cosh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, x^2\right) + \frac{1}{2}(b\sinh(a))\text{Subst}\Bigg(\int\\
&= -\frac{\cosh(a+bx^2)}{2x^2} + \frac{1}{2}b\text{Chi}(bx^2)\sinh(a) + \frac{1}{2}b\cosh(a)\text{Shi}(bx^2)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.90

$$\frac{1}{2}\left(-\frac{\cosh(a+bx^2)}{x^2} + b\text{Chi}(bx^2)\sinh(a) + b\cosh(a)\text{Shi}(bx^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]/x^3,x]
```

```
[Out] (-(Cosh[a + b*x^2]/x^2) + b*CoshIntegral[b*x^2]*Sinh[a] + b*Cosh[a]*SinhInt
egral[b*x^2])/2
```

Maple [A]
time = 0.44, size = 58, normalized size = 1.38

method	result
risch	$-\frac{e^{-a}e^{-x^2 b}}{4x^2} + \frac{e^{-a} b \,\text{expIntegral}(1, x^2 b)}{4} - \frac{e^{a} e^{x^2 b}}{4x^2} - \frac{e^{a} b \,\text{expIntegral}(1, -x^2 b)}{4}$
meijerg	$\frac{i\cosh(a)\sqrt{\pi}\, b\left(\frac{4i\cosh(x^2 b)}{b x^2\sqrt{\pi}} - \frac{4i\,\text{hyperbolicSineIntegral}(x^2 b)}{\sqrt{\pi}}\right)}{8} + \frac{\sinh(a)\sqrt{\pi}\, b\left(\frac{4}{\sqrt{\pi}} - \frac{4\sinh(x^2 b)}{\sqrt{\pi}\, x^2 b} + \frac{4\,\text{hyperbolicCosineIntegral}(x^2 b)}{\sqrt{\pi}}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*exp(-a)/x^2*exp(-x^2*b)+1/4*exp(-a)*b*Ei(1,x^2*b)-1/4*exp(a)*exp(x^2*b
)/x^2-1/4*exp(a)*b*Ei(1,-x^2*b)
```

Maxima [A]
time = 0.31, size = 40, normalized size = 0.95

$$-\frac{1}{4}\left(\mathrm{Ei}\left(-bx^2\right)e^{(-a)} - \mathrm{Ei}\left(bx^2\right)e^{a}\right)b - \frac{\cosh\left(bx^2+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(Ei(-b*x^2)*e^(-a) - Ei(b*x^2)*e^a)*b - 1/2*cosh(b*x^2 + a)/x^2
```

Fricas [A]
time = 0.36, size = 71, normalized size = 1.69

$$\frac{\left(bx^2\mathrm{Ei}(bx^2) - bx^2\mathrm{Ei}(-bx^2)\right)\cosh\left(a\right) + \left(bx^2\mathrm{Ei}(bx^2) + bx^2\mathrm{Ei}(-bx^2)\right)\sinh\left(a\right) - 2\,\cosh\left(bx^2+a\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^2*Ei(b*x^2) - b*x^2*Ei(-b*x^2))*cosh(a) + (b*x^2*Ei(b*x^2) + b*x^
2*Ei(-b*x^2))*sinh(a) - 2*cosh(b*x^2 + a))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)/x**3,x)
```

```
[Out] Integral(cosh(a + b*x**2)/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(36) = 72.
time = 0.40, size = 108, normalized size = 2.57

$$-\frac{(bx^2+a)b^2\mathrm{Ei}(-bx^2)\,e^{(-a)} - ab^2\mathrm{Ei}(-bx^2)\,e^{(-a)} - (bx^2+a)b^2\mathrm{Ei}(bx^2)\,e^{a} + ab^2\mathrm{Ei}(bx^2)\,e^{a} + b^2e^{(bx^2+a)} + b^2e^{(-bx^2-a)}}{4\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*((b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) - a*b^2*Ei(-b*x^2)*e^(-a) - (b*x^2
+ a)*b^2*Ei(b*x^2)*e^a + a*b^2*Ei(b*x^2)*e^a + b^2*e^(b*x^2 + a) + b^2*e^(-
b*x^2 - a))/(b^2*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(b\,x^2+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)/x^3,x)
```

```
[Out] int(cosh(a + b*x^2)/x^3, x)
```

3.8 $\int x^3 \cosh^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=51

$$\frac{x^4}{8}-\frac{\cosh^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}$$

```
[Out] 1/8*x^4-1/8*cosh(b*x^2+a)^2/b^2+1/4*x^2*cosh(b*x^2+a)*sinh(b*x^2+a)/b
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {5429, 3391, 30}

$$-\frac{\cosh^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}+\frac{x^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x^2]^2,x]
```

```
[Out] x^4/8 - Cosh[a + b*x^2]^2/(8*b^2) + (x^2*Cosh[a + b*x^2]*Sinh[a + b*x^2])/(
4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \cosh^2\left(a+bx^2\right)\,dx = \frac{1}{2}\mathrm{Subst}\left(\int x\cosh^2(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{\cosh^2\left(a+bx^2\right)}{8b^2} + \frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b} + \frac{1}{4}\mathrm{Subst}\left(\int x\,dx, x, x^2\right)$$
$$= \frac{x^4}{8} - \frac{\cosh^2\left(a+bx^2\right)}{8b^2} + \frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}$$

Mathematica [A]
time = 0.08, size = 40, normalized size = 0.78

$$-\frac{\cosh\left(2(a+bx^2)\right) - 2bx^2\left(bx^2+\sinh\left(2(a+bx^2)\right)\right)}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x^2]^2,x]
```

```
[Out] -1/16*(Cosh[2*(a + b*x^2)] - 2*b*x^2*(b*x^2 + Sinh[2*(a + b*x^2)]))/b^2
```

Maple [A]
time = 1.53, size = 55, normalized size = 1.08

method	result	size
risch	$\frac{x^4}{8} + \frac{(2x^2b-1)\mathrm{e}^{2x^2b+2a}}{32b^2} - \frac{(2x^2b+1)\mathrm{e}^{-2x^2b-2a}}{32b^2}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^4+1/32*(2*b*x^2-1)/b^2*exp(2*b*x^2+2*a)-1/32*(2*b*x^2+1)/b^2*exp(-2*b
*x^2-2*a)
```

Maxima [A]
time = 0.26, size = 59, normalized size = 1.16

$$\frac{1}{8}\,x^4 + \frac{\left(2\,bx^2e^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx^2)}}{32\,b^2} - \frac{(2\,bx^2+1)e^{(-2\,bx^2-2\,a)}}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x^4 + 1/32*(2*b*x^2*e^(2*a) - e^(2*a))*e^(2*b*x^2)/b^2 - 1/32*(2*b*x^2
+ 1)*e^(-2*b*x^2 - 2*a)/b^2
```

Fricas [A]
time = 0.35, size = 60, normalized size = 1.18

$$\frac{2\,b^2x^4 + 4\,bx^2\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right) - \cosh\left(bx^2+a\right)^2 - \sinh\left(bx^2+a\right)^2}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*b^2*x^4 + 4*b*x^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) - cosh(b*x^2 + a)
^2 - sinh(b*x^2 + a)^2)/b^2
```

Sympy [A]
time = 0.28, size = 78, normalized size = 1.53

$$\begin{cases} -\frac{x^4\sinh^2\left(a+bx^2\right)}{8} + \frac{x^4\cosh^2\left(a+bx^2\right)}{8} + \frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b} - \frac{\cosh^2\left(a+bx^2\right)}{8b^2} & \text{for } b \neq 0 \\ \frac{x^4\cosh^2\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x**2+a)**2,x)
```

```
[Out] Piecewise((-x**4*sinh(a + b*x**2)**2/8 + x**4*cosh(a + b*x**2)**2/8 + x**2*
sinh(a + b*x**2)*cosh(a + b*x**2)/(4*b) - cosh(a + b*x**2)**2/(8*b**2), Ne(
b, 0)), (x**4*cosh(a)**2/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(45) = 90.
time = 0.40, size = 142, normalized size = 2.78

$$\frac{4\left(bx^2+a\right)^2 + 2\left(bx^2+a\right)e^{\left(2\,bx^2+2\,a\right)} - 2\left(bx^2+a\right)e^{\left(-2\,bx^2-2\,a\right)} - e^{\left(2\,bx^2+2\,a\right)} - e^{\left(-2\,bx^2-2\,a\right)}}{32\,b^2} - \frac{4\left(bx^2+a\right)a + ae^{\left(2\,bx^2+2\,a\right)} - \left(2\,ae^{\left(2\,bx^2+2\,a\right)} + a\right)e^{\left(-2\,bx^2-2\,a\right)}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*(4*(b*x^2 + a)^2 + 2*(b*x^2 + a)*e^(2*b*x^2 + 2*a) - 2*(b*x^2 + a)*e^(
-2*b*x^2 - 2*a) - e^(2*b*x^2 + 2*a) - e^(-2*b*x^2 - 2*a))/b^2 - 1/16*(4*(b*
x^2 + a)*a + a*e^(2*b*x^2 + 2*a) - (2*a*e^(2*b*x^2 + 2*a) + a)*e^(-2*b*x^2
- 2*a))/b^2
```

Mupad [B]
time = 0.11, size = 42, normalized size = 0.82

$$\frac{x^4}{8} - \frac{\frac{\cosh\left(2\,b\,x^2+2\,a\right)}{16} - \frac{b\,x^2\,\sinh\left(2\,b\,x^2+2\,a\right)}{8}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x^2)^2,x)
```

```
[Out] x^4/8 - (cosh(2*a + 2*b*x^2)/16 - (b*x^2*sinh(2*a + 2*b*x^2))/8)/b^2
```

3.9 $\int x^2 \cosh^2(a+bx^2)\,dx$

Optimal. Leaf size=99

$$\frac{x^3}{6}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{e^{2a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{32b^{3/2}}+\frac{x\sinh(2a+2bx^2)}{8b}$$

```
[Out] 1/6*x^3+1/8*x*sinh(2*b*x^2+2*a)/b+1/64*erf(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1
/2)/b^(3/2)/exp(2*a)-1/64*exp(2*a)*erfi(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)
/b^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5449, 5433, 5406, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\,e^{-2a}\mathrm{Erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\,e^{2a}\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{32b^{3/2}}+\frac{x\sinh(2a+2bx^2)}{8b}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x^2]^2,x]
```

```
[Out] x^3/6 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)*E^(2*a)) - (E^(2*a)
*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)) + (x*Sinh[2*a + 2*b*x^2])
/(8*b)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\cosh^2\left(a+bx^2\right)\,dx &= \int\left(\frac{x^2}{2}+\frac{1}{2}x^2\cosh\left(2a+2bx^2\right)\right)\,dx\\&=\frac{x^3}{6}+\frac{1}{2}\int x^2\cosh\left(2a+2bx^2\right)\,dx\\&=\frac{x^3}{6}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}-\frac{\int\sinh\left(2a+2bx^2\right)\,dx}{8b}\\&=\frac{x^3}{6}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}+\frac{\int e^{-2a-2bx^2}\,dx}{16b}-\frac{\int e^{2a+2bx^2}\,dx}{16b}\\&=\frac{x^3}{6}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)}{32b^{3/2}}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}\end{aligned}$$

Mathematica [**A**]
time = 0.16, size = 101, normalized size = 1.02

$$\frac{3\sqrt{2\pi}\operatorname{Erf}\left(\sqrt{2}\sqrt{b}\,x\right)(\cosh(2a)-\sinh(2a))-3\sqrt{2\pi}\operatorname{Erfi}\left(\sqrt{2}\sqrt{b}\,x\right)(\cosh(2a)+\sinh(2a))+8\sqrt{b}\,x(4bx^2+3\sinh\left(2(a+bx^2)\right))}{192b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x^2]^2,x]
```

```
[Out] (3*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2*a]) - 3*Sqrt[2*Pi]
*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]) + 8*Sqrt[b]*x*(4*b*x^2 + 3
*Sinh[2*(a + b*x^2)]))/(192*b^(3/2))
```

Maple [**A**]
time = 1.52, size = 90, normalized size = 0.91

method	result	size

risch	$\frac{x^{3}}{6}-\frac{\mathrm{e}^{-2a} x\,\mathrm{e}^{-2x^{2} b}}{16 b}+\frac{\mathrm{e}^{-2a} \sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{64 b^{\frac{3}{2}}}+\frac{\mathrm{e}^{2a} x\,\mathrm{e}^{2x^{2} b}}{16 b}-\frac{\mathrm{e}^{2a} \sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2b}\,x\right)}{32 b \sqrt{-2b}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^3-1/16*exp(-2*a)/b*x*exp(-2*x^2*b)+1/64*exp(-2*a)/b^(3/2)*Pi^(1/2)*2^
(1/2)*erf(x*2^(1/2)*b^(1/2))+1/16*exp(2*a)/b*x*exp(2*x^2*b)-1/32*exp(2*a)/b
*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

Maxima [A]
time = 0.48, size = 95, normalized size = 0.96

$$\frac{1}{6}\,x^{3}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{-b}\,x\right)e^{(2\,a)}}{64\,\sqrt{-b}\,b}+\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)e^{(-2\,a)}}{64\,b^{\frac{3}{2}}}+\frac{xe^{(2\,bx^{2}+2\,a)}}{16\,b}-\frac{xe^{(-2\,bx^{2}-2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)*b
) + 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16*x*
e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(71) = 142.
time = 0.39, size = 427, normalized size = 4.31

$$\frac{32\,b^{2}x^{3}\cosh\left(bx^{2}+a\right)^{2}+12\,bx\cosh\left(bx^{2}+a\right)^{4}+48\,bx\cosh\left(bx^{2}+a\right)\sinh\left(bx^{2}+a\right)^{3}+12\,bx\sinh\left(bx^{2}+a\right)^{4}+3\,\sqrt{2}\,\sqrt{\pi}\left(\cosh\left(bx^{2}+a\right)^{2}\cosh\left(2\,a\right)+\left(\cosh\left(2\,a\right)+\sinh\left(2\,a\right)\right)\sinh\left(bx^{2}+a\right)^{2}+\cosh\left(bx^{2}+a\right)^{2}\sinh\left(2\,a\right)+2\left(\cosh\left(bx^{2}+a\right)\cosh\left(2\,a\right)+\cosh\left(bx^{2}+a\right)\sinh\left(2\,a\right)\right)\sinh\left(bx^{2}+a\right)\right)\sqrt{-b}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{-b}\,x\right)+3\,\sqrt{2}\,\sqrt{\pi}\left(\cosh\left(bx^{2}+a\right)^{2}\cosh\left(2\,a\right)+\left(\cosh\left(2\,a\right)-\sinh\left(2\,a\right)\right)\sinh\left(bx^{2}+a\right)^{2}-\cosh\left(bx^{2}+a\right)^{2}\sinh\left(2\,a\right)+2\left(\cosh\left(bx^{2}+a\right)\cosh\left(2\,a\right)-\cosh\left(bx^{2}+a\right)\sinh\left(2\,a\right)\right)\sinh\left(bx^{2}+a\right)\right)\sqrt{b}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)+8\left(4\,b^{2}x^{3}+9\,bx\cosh\left(bx^{2}+a\right)^{2}\right)\sinh\left(bx^{2}+a\right)^{2}-12\,bx+16\left(4\,b^{2}x^{3}\cosh\left(bx^{2}+a\right)+3\,bx\cosh\left(bx^{2}+a\right)^{3}\right)\sinh\left(bx^{2}+a\right)}{192\left(b^{2}\cosh\left(bx^{2}+a\right)^{2}+2\,b^{2}\cosh\left(bx^{2}+a\right)\sinh\left(bx^{2}+a\right)+b^{2}\sinh\left(bx^{2}+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(32*b^2*x^3*cosh(b*x^2 + a)^2 + 12*b*x*cosh(b*x^2 + a)^4 + 48*b*x*cos
h(b*x^2 + a)*sinh(b*x^2 + a)^3 + 12*b*x*sinh(b*x^2 + a)^4 + 3*sqrt(2)*sqrt(
pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) + sinh(2*a))*sinh(b*x^2 + a)^
2 + cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2*a) + cosh(b*x^2
 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(2)*sqrt(-b)*x) + 3*sqrt
(2)*sqrt(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) - sinh(2*a))*sinh(b*
x^2 + a)^2 - cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2*a) - c
osh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x) +
 8*(4*b^2*x^3 + 9*b*x*cosh(b*x^2 + a)^2)*sinh(b*x^2 + a)^2 - 12*b*x + 16*(4
*b^2*x^3*cosh(b*x^2 + a) + 3*b*x*cosh(b*x^2 + a)^3)*sinh(b*x^2 + a))/(b^2*c
osh(b*x^2 + a)^2 + 2*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) + b^2*sinh(b*x^2 +
 a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh^2\left(a+bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*cosh(a + b*x**2)**2, x)
```

Giac [A]
time = 0.41, size = 97, normalized size = 0.98

$$\frac{1}{6}\,x^3 + \frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{2}\,\sqrt{-b}\,x\right) e^{(2\,a)}}{64\,\sqrt{-b}\,b} - \frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{2}\,\sqrt{b}\,x\right) e^{(-2\,a)}}{64\,b^{\frac{3}{2}}} + \frac{xe^{(2\,bx^2+2\,a)}}{16\,b} - \frac{xe^{(-2\,bx^2-2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)*
b) - 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16*
x*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\cosh\left(b\,x^2+a\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x^2)^2,x)
```

```
[Out] int(x^2*cosh(a + b*x^2)^2, x)
```

3.10 $\int x \cosh^2(a+bx^2)\, dx$

Optimal. Leaf size=31

$$\frac{x^2}{4}+\frac{\cosh(a+bx^2)\sinh(a+bx^2)}{4b}$$

```
[Out] 1/4*x^2+1/4*cosh(b*x^2+a)*sinh(b*x^2+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5429, 2715, 8}

$$\frac{\sinh(a+bx^2)\cosh(a+bx^2)}{4b}+\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x^2]^2,x]
```

```
[Out] x^2/4 + (Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int x\cosh^2\left(a+bx^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int\cosh^2(a+bx)\,dx,x,x^2\right)$$
$$= \frac{\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b} + \frac{1}{4}\text{Subst}\left(\int 1\,dx,x,x^2\right)$$
$$= \frac{x^2}{4} + \frac{\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.87

$$\frac{2(a+bx^2)+\sinh\left(2(a+bx^2)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x^2]^2,x]
[Out] (2*(a + b*x^2) + Sinh[2*(a + b*x^2)])/(8*b)
```

Maple [A]
time = 0.65, size = 34, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\frac{\cosh\left(x^2b+a\right)\sinh\left(x^2b+a\right)}{2}+\frac{x^2b}{2}+\frac{a}{2}}{2b}$	34
default	$\frac{\frac{\cosh\left(x^2b+a\right)\sinh\left(x^2b+a\right)}{2}+\frac{x^2b}{2}+\frac{a}{2}}{2b}$	34
risch	$\frac{x^2}{4}+\frac{\mathrm{e}^{2x^2b+2a}}{16b}-\frac{\mathrm{e}^{-2x^2b-2a}}{16b}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2/b*(1/2*cosh(b*x^2+a)*sinh(b*x^2+a)+1/2*x^2*b+1/2*a)
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.23

$$\frac{1}{4}\,x^2+\frac{e^{\left(2\,bx^2+2\,a\right)}}{16\,b}-\frac{e^{\left(-2\,bx^2-2\,a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2 + 1/16*e^(2*b*x^2 + 2*a)/b - 1/16*e^(-2*b*x^2 - 2*a)/b
```

Fricas [A]
time = 0.40, size = 28, normalized size = 0.90

$$\frac{bx^2 + \cosh\left(bx^2 + a\right)\sinh\left(bx^2 + a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b*x^2 + cosh(b*x^2 + a)*sinh(b*x^2 + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.
time = 0.13, size = 60, normalized size = 1.94

$$\begin{cases} -\frac{x^2\sinh^2\left(a+bx^2\right)}{4} + \frac{x^2\cosh^2\left(a+bx^2\right)}{4} + \frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b} & \text{for}\ b \neq 0 \\ \frac{x^2\cosh^2\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x**2+a)**2,x)
```

```
[Out] Piecewise((-x**2*sinh(a + b*x**2)**2/4 + x**2*cosh(a + b*x**2)**2/4 + sinh(
a + b*x**2)*cosh(a + b*x**2)/(4*b), Ne(b, 0)), (x**2*cosh(a)**2/2, True))
```

Giac [A]
time = 0.39, size = 54, normalized size = 1.74

$$\frac{4\,bx^2 - \left(2\,e^{\left(2\,bx^2+2\,a\right)} + 1\right)e^{\left(-2\,bx^2-2\,a\right)} + 4\,a + e^{\left(2\,bx^2+2\,a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*b*x^2 - (2*e^(2*b*x^2 + 2*a) + 1)*e^(-2*b*x^2 - 2*a) + 4*a + e^(2*b
*x^2 + 2*a))/b
```

Mupad [B]
time = 0.06, size = 22, normalized size = 0.71

$$\frac{\sinh(2\,b\,x^2 + 2\,a)}{8\,b} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x^2)^2,x)
```

```
[Out] sinh(2*a + 2*b*x^2)/(8*b) + x^2/4
```

3.11 $\int \cosh^2(a+bx^2)\,dx$

Optimal. Leaf size=78

$$\frac{x}{2}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{8\sqrt{b}}+\frac{e^{2a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{8\sqrt{b}}$$

```
[Out] 1/2*x+1/16*erf(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/exp(2*a)/b^(1/2)+1/16*ex
p(2*a)*erfi(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {5409, 5407, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\,e^{-2a}\mathrm{Erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{8\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\,e^{2a}\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{8\sqrt{b}}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]^2,x]
```

```
[Out] x/2 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b]*E^(2*a)) + (E^(2*a)*Sq
rt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5409

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \cosh^2\left(a+bx^2\right)\,dx &= \int \left(\frac{1}{2}+\frac{1}{2}\cosh\left(2a+2bx^2\right)\right)\,dx\\ &= \frac{x}{2}+\frac{1}{2}\int \cosh\left(2a+2bx^2\right)\,dx\\ &= \frac{x}{2}+\frac{1}{4}\int e^{-2a-2bx^2}\,dx+\frac{1}{4}\int e^{2a+2bx^2}\,dx\\ &= \frac{x}{2}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{8\sqrt{b}}+\frac{e^{2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)}{8\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 86, normalized size = 1.10

$$\frac{4\sqrt{2}\,\sqrt{b}\,x+\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)(\cosh(2a)-\sinh(2a))+\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)(\cosh(2a)+\sinh(2a))}{8\sqrt{2}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]^2,x]
```

```
[Out] (4*Sqrt[2]*Sqrt[b]*x + Sqrt[Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2*
a]) + Sqrt[Pi]*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]))/(8*Sqrt[2]*
Sqrt[b])
```

Maple [A]
time = 1.24, size = 51, normalized size = 0.65

method	result	size
risch	$\frac{x}{2}+\frac{\mathrm{e}^{-2a}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{16\sqrt{b}}+\frac{\mathrm{e}^{2a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2b}\,x\right)}{8\sqrt{-2b}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/16*exp(-2*a)*Pi^(1/2)*2^(1/2)/b^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/8*ex
p(2*a)*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

Maxima [A]
time = 0.48, size = 56, normalized size = 0.72

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{-b}\,x\right)e^{(2\,a)}}{16\,\sqrt{-b}}+\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)e^{(-2\,a)}}{16\,\sqrt{b}}+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) + 1/16*sqrt(
2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) + 1/2*x
```

Fricas [A]
time = 0.44, size = 73, normalized size = 0.94

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{-b}\,(\cosh(2\,a)+\sinh(2\,a))\operatorname{erf}\left(\sqrt{2}\,\sqrt{-b}\,x\right)-\sqrt{2}\,\sqrt{\pi}\,\sqrt{b}\,(\cosh(2\,a)-\sinh(2\,a))\operatorname{erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)-8\,bx}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*sqrt(-b)*(cosh(2*a) + sinh(2*a))*erf(sqrt(2)*sqrt(-
b)*x) - sqrt(2)*sqrt(pi)*sqrt(b)*(cosh(2*a) - sinh(2*a))*erf(sqrt(2)*sqrt(b
)*x) - 8*b*x)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh^2\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)**2,x)
```

```
[Out] Integral(cosh(a + b*x**2)**2, x)
```

Giac [A]
time = 0.41, size = 58, normalized size = 0.74

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{2}\,\sqrt{-b}\,x\right)e^{(2\,a)}}{16\,\sqrt{-b}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{2}\,\sqrt{b}\,x\right)e^{(-2\,a)}}{16\,\sqrt{b}}+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) - 1/16*sqr
t(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) + 1/2*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(b\,x^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)^2,x)
```

```
[Out] int(cosh(a + b*x^2)^2, x)
```

3.12 $\int \frac{\cosh^2(a+bx^2)}{x} \, dx$

Optimal. Leaf size=37

$$\frac{1}{4}\cosh(2a)\mathrm{Chi}\left(2bx^2\right) + \frac{\log(x)}{2} + \frac{1}{4}\sinh(2a)\mathrm{Shi}\left(2bx^2\right)$$

```
[Out] 1/4*Chi(2*b*x^2)*cosh(2*a)+1/2*ln(x)+1/4*Shi(2*b*x^2)*sinh(2*a)
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5449, 5427, 5425, 5424}

$$\frac{1}{4}\cosh(2a)\mathrm{Chi}\left(2bx^2\right) + \frac{1}{4}\sinh(2a)\mathrm{Shi}\left(2bx^2\right) + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2])/4 + Log[x]/2 + (Sinh[2*a]*SinhIntegral[2*
b*x^2])/4
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(a+bx^2)}{x}\,dx &= \int \left(\frac{1}{2x}+\frac{\cosh(2a+2bx^2)}{2x}\right)dx \\
&= \frac{\log(x)}{2}+\frac{1}{2}\int \frac{\cosh(2a+2bx^2)}{x}\,dx \\
&= \frac{\log(x)}{2}+\frac{1}{2}\cosh(2a)\int \frac{\cosh(2bx^2)}{x}\,dx+\frac{1}{2}\sinh(2a)\int \frac{\sinh(2bx^2)}{x}\,dx \\
&= \frac{1}{4}\cosh(2a)\mathrm{Chi}(2bx^2)+\frac{\log(x)}{2}+\frac{1}{4}\sinh(2a)\mathrm{Shi}(2bx^2)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(\cosh(2a)\mathrm{Chi}(2bx^2)+2\log(x)+\sinh(2a)\mathrm{Shi}(2bx^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2] + 2*Log[x] + Sinh[2*a]*SinhIntegral[2*b*x^
2])/4
```

Maple [A]
time = 1.23, size = 34, normalized size = 0.92

method	result	size
risch	$\frac{\ln(x)}{2}-\frac{e^{-2a}\,\mathrm{expIntegral}(1,2x^2b)}{8}-\frac{e^{2a}\,\mathrm{expIntegral}(1,-2x^2b)}{8}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)-1/8*exp(-2*a)*Ei(1,2*x^2*b)-1/8*exp(2*a)*Ei(1,-2*x^2*b)
```

Maxima [A]
time = 0.30, size = 31, normalized size = 0.84

$$\frac{1}{8}\,\mathrm{Ei}(2\,bx^2)\,e^{(2\,a)}+\frac{1}{8}\,\mathrm{Ei}(-2\,bx^2)\,e^{(-2\,a)}+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) + 1/2*log(x)
```

Fricas [A]
time = 0.36, size = 49, normalized size = 1.32

$$\frac{1}{8}\left(\mathrm{Ei}\left(2\,bx^{2}\right)+\mathrm{Ei}\left(-2\,bx^{2}\right)\right)\cosh\left(2\,a\right)+\frac{1}{8}\left(\mathrm{Ei}\left(2\,bx^{2}\right)-\mathrm{Ei}\left(-2\,bx^{2}\right)\right)\sinh\left(2\,a\right)+\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2/x,x, algorithm="fricas")
[Out] 1/8*(Ei(2*b*x^2) + Ei(-2*b*x^2))*cosh(2*a) + 1/8*(Ei(2*b*x^2) - Ei(-2*b*x^2
))*sinh(2*a) + 1/2*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^{2}\left(a+bx^{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)**2/x,x)
[Out] Integral(cosh(a + b*x**2)**2/x, x)
```

Giac [A]
time = 0.41, size = 35, normalized size = 0.95

$$\frac{1}{8}\,\mathrm{Ei}\left(2\,bx^{2}\right)e^{\left(2\,a\right)}+\frac{1}{8}\,\mathrm{Ei}\left(-2\,bx^{2}\right)e^{\left(-2\,a\right)}+\frac{1}{4}\,\log\left(bx^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2/x,x, algorithm="giac")
[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) + 1/4*log(b*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{{\mathrm{cosh}\left(b\,x^2+a\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)^2/x,x)
[Out] int(cosh(a + b*x^2)^2/x, x)
```

3.13 $\int \frac{\cosh^2(a+bx^2)}{x^2} \, dx$

Optimal. Leaf size=88

$$-\frac{\cosh^2\left(a+bx^2\right)}{x}-\frac{1}{2}\sqrt{b}\,e^{-2a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{b}\,e^{2a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)$$

```
[Out] -cosh(b*x^2+a)^2/x-1/4*erf(x*2^(1/2)*b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/exp(
2*a)+1/4*exp(2*a)*erfi(x*2^(1/2)*b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5439, 5736, 5422, 5406, 2235, 2236}

$$-\frac{1}{2}\sqrt{\frac{\pi}{2}}\,e^{-2a}\sqrt{b}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\,e^{2a}\sqrt{b}\,\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)-\frac{\cosh^2\left(a+bx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]^2/x^2,x]
```

```
[Out] -(Cosh[a + b*x^2]^2/x) - (Sqrt[b]*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(2*E^(
2*a)) + (Sqrt[b]*E^(2*a)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/2
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5422

```
Int[((a_.) + (b_.)*Sinh[u_])^(p_.), x_Symbol] :> Int[(a + b*Sinh[ExpandToSu
m[u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatc
```

```
hQ[u, x]
```

Rule 5439

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[-Cosh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Dist[b*n*(p/(n - 1)), Int[Cosh[a + b
*x^n]^(p - 1)*Sinh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n, p
] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

Rule 5736

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\cosh^2\left(a+bx^2\right)}{x}+(4b)\int \cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)\,dx\\ &= -\frac{\cosh^2\left(a+bx^2\right)}{x}+(2b)\int \sinh\left(2\left(a+bx^2\right)\right)\,dx\\ &= -\frac{\cosh^2\left(a+bx^2\right)}{x}+(2b)\int \sinh\left(2a+2bx^2\right)\,dx\\ &= -\frac{\cosh^2\left(a+bx^2\right)}{x}-b\int e^{-2a-2bx^2}\,dx+b\int e^{2a+2bx^2}\,dx\\ &= -\frac{\cosh^2\left(a+bx^2\right)}{x}-\frac{1}{2}\sqrt{b}\,e^{-2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{b}\,e^{2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)\end{aligned}$$

Mathematica [A]
time = 0.18, size = 94, normalized size = 1.07

$$\frac{-4\cosh^2\left(a+bx^2\right)+\sqrt{b}\,\sqrt{2\pi}\,x\mathrm{Erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)\left(-\cosh(2a)+\sinh(2a)\right)+\sqrt{b}\,\sqrt{2\pi}\,x\mathrm{Erfi}\left(\sqrt{2}\,\sqrt{b}\,x\right)\left(\cosh(2a)+\sinh(2a)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]^2/x^2,x]
```

```
[Out] (-4*Cosh[a + b*x^2]^2 + Sqrt[b]*Sqrt[2*Pi]*x*Erf[Sqrt[2]*Sqrt[b]*x]*(-Cosh[
2*a] + Sinh[2*a]) + Sqrt[b]*Sqrt[2*Pi]*x*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a]
 + Sinh[2*a]))/(4*x)
```

Maple [A]
time = 1.36, size = 86, normalized size = 0.98

method	result	size
risch	$-\frac{1}{2x}-\frac{\mathrm{e}^{-2a}\mathrm{e}^{-2x^2b}}{4x}-\frac{\mathrm{e}^{-2a}\sqrt{b}\,\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{4}-\frac{\mathrm{e}^{2a}\mathrm{e}^{2x^2b}}{4x}+\frac{\mathrm{e}^{2a}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2b}\,x\right)}{2\sqrt{-2b}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x-1/4*exp(-2*a)/x*exp(-2*x^2*b)-1/4*exp(-2*a)*b^(1/2)*Pi^(1/2)*2^(1/2)
*erf(x*2^(1/2)*b^(1/2))-1/4*exp(2*a)/x*exp(2*x^2*b)+1/2*exp(2*a)*b*Pi^(1/2)
/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

Maxima [A]
time = 0.32, size = 61, normalized size = 0.69

$$-\frac{\sqrt{2}\,\sqrt{bx^2}\,e^{(-2\,a)}\Gamma\left(-\frac{1}{2},2\,bx^2\right)}{8\,x}-\frac{\sqrt{2}\,\sqrt{-bx^2}\,e^{(2\,a)}\Gamma\left(-\frac{1}{2},-2\,bx^2\right)}{8\,x}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(b*x^2)*e^(-2*a)*gamma(-1/2, 2*b*x^2)/x - 1/8*sqrt(2)*sqrt
(-b*x^2)*e^(2*a)*gamma(-1/2, -2*b*x^2)/x - 1/2/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(64) = 128.
time = 0.41, size = 394, normalized size = 4.48

$$-\frac{\cosh\left(bx^2+a\right)^4+4\,\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^3+\sinh\left(bx^2+a\right)^4+\sqrt{2}\,\sqrt{\pi}\left(x\cosh\left(bx^2+a\right)^2\cosh\left(2\,a\right)+x\cosh\left(bx^2+a\right)^2\sinh\left(2\,a\right)+\left(x\cosh\left(2\,a\right)+x\sinh\left(2\,a\right)\right)\sinh\left(bx^2+a\right)^2+2\left(x\cosh\left(bx^2+a\right)\cosh\left(2\,a\right)+x\cosh\left(bx^2+a\right)\sinh\left(2\,a\right)\right)\sinh\left(bx^2+a\right)\right)\sqrt{-b}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{-b}\,x\right)+\sqrt{2}\,\sqrt{\pi}\left(x\cosh\left(bx^2+a\right)^2\cosh\left(2\,a\right)-x\cosh\left(bx^2+a\right)^2\sinh\left(2\,a\right)+\left(x\cosh\left(2\,a\right)-x\sinh\left(2\,a\right)\right)\sinh\left(bx^2+a\right)^2+2\left(x\cosh\left(bx^2+a\right)\cosh\left(2\,a\right)-x\cosh\left(bx^2+a\right)\sinh\left(2\,a\right)\right)\sinh\left(bx^2+a\right)\right)\sqrt{b}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{b}\,x\right)+2\left(3\,\cosh\left(bx^2+a\right)^2+1\right)\sinh\left(bx^2+a\right)^2+2\,\cosh\left(bx^2+a\right)^2+4\left(\cosh\left(bx^2+a\right)^3+\cosh\left(bx^2+a\right)\right)\sinh\left(bx^2+a\right)+1}{4\left(x\cosh\left(bx^2+a\right)^2+2\,x\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)+x\sinh\left(bx^2+a\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cosh(b*x^2 + a)^4 + 4*cosh(b*x^2 + a)*sinh(b*x^2 + a)^3 + sinh(b*x^2
+ a)^4 + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a) + x*cosh(b*x^2 + a
)^2*sinh(2*a) + (x*cosh(2*a) + x*sinh(2*a))*sinh(b*x^2 + a)^2 + 2*(x*cosh(b
*x^2 + a)*cosh(2*a) + x*cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b
)*erf(sqrt(2)*sqrt(-b)*x) + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a)
 - x*cosh(b*x^2 + a)^2*sinh(2*a) + (x*cosh(2*a) - x*sinh(2*a))*sinh(b*x^2 +
 a)^2 + 2*(x*cosh(b*x^2 + a)*cosh(2*a) - x*cosh(b*x^2 + a)*sinh(2*a))*sinh(
b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x) + 2*(3*cosh(b*x^2 + a)^2 + 1)*si
nh(b*x^2 + a)^2 + 2*cosh(b*x^2 + a)^2 + 4*(cosh(b*x^2 + a)^3 + cosh(b*x^2 +
 a))*sinh(b*x^2 + a) + 1)/(x*cosh(b*x^2 + a)^2 + 2*x*cosh(b*x^2 + a)*sinh(b
*x^2 + a) + x*sinh(b*x^2 + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)**2/x**2,x)
```

```
[Out] Integral(cosh(a + b*x**2)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x^2 + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(b\,x^2+a\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)^2/x^2,x)
```

```
[Out] int(cosh(a + b*x^2)^2/x^2, x)
```

3.14 $\int \frac{\cosh^2(a+bx^2)}{x^3} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{4x^2}-\frac{\cosh\left(2(a+bx^2)\right)}{4x^2}+\frac{1}{2}b\text{Chi}\left(2bx^2\right)\sinh(2a)+\frac{1}{2}b\cosh(2a)\text{Shi}\left(2bx^2\right)$$

```
[Out] -1/4/x^2-1/4*cosh(2*b*x^2+2*a)/x^2+1/2*b*cosh(2*a)*Shi(2*b*x^2)+1/2*b*Chi(2
*b*x^2)*sinh(2*a)
```

Rubi [A]
time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5449, 5429, 3378, 3384, 3379, 3382}

$$\frac{1}{2}b\sinh(2a)\text{Chi}\left(2bx^2\right)+\frac{1}{2}b\cosh(2a)\text{Shi}\left(2bx^2\right)-\frac{\cosh\left(2(a+bx^2)\right)}{4x^2}-\frac{1}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]^2/x^3,x]
```

```
[Out] -1/4*1/x^2 - Cosh[2*(a + b*x^2)]/(4*x^2) + (b*CoshIntegral[2*b*x^2]*Sinh[2*
a])/2 + (b*Cosh[2*a]*SinhIntegral[2*b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2\left(a+bx^2\right)}{x^3}\,dx &= \int \left(\frac{1}{2x^3}+\frac{\cosh\left(2a+2bx^2\right)}{2x^3}\right)\,dx \\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{\cosh\left(2a+2bx^2\right)}{x^3}\,dx \\
&= -\frac{1}{4x^2}+\frac{1}{4}\text{Subst}\left(\int \frac{\cosh(2a+2bx)}{x^2}\,dx,x,x^2\right) \\
&= -\frac{1}{4x^2}-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{2}b\text{Subst}\left(\int \frac{\sinh(2a+2bx)}{x}\,dx,x,x^2\right) \\
&= -\frac{1}{4x^2}-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{2}(b\cosh(2a))\text{Subst}\left(\int \frac{\sinh(2bx)}{x}\,dx,x,x^2\right)+\frac{1}{2}(b\,\mathrm{si} \\
&= -\frac{1}{4x^2}-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{2}b\text{Chi}\left(2bx^2\right)\sinh(2a)+\frac{1}{2}b\cosh(2a)\text{Shi}\left(2bx^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.81

$$\frac{1}{2}\left(-\frac{\cosh^2\left(a+bx^2\right)}{x^2}+b\text{Chi}\left(2bx^2\right)\sinh(2a)+b\cosh(2a)\text{Shi}\left(2bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]^2/x^3,x]
```

```
[Out] (-(Cosh[a + b*x^2]^2/x^2) + b*CoshIntegral[2*b*x^2]*Sinh[2*a] + b*Cosh[2*a]
*SinhIntegral[2*b*x^2])/2
```

Maple [A]
time = 1.23, size = 69, normalized size = 1.21

method	result	size
risch	$-\frac{1}{4x^2}-\frac{e^{-2a}e^{-2x^2b}}{8x^2}+\frac{e^{-2a}b\,\mathrm{expIntegral}(1,2x^2b)}{4}-\frac{e^{2a}e^{2x^2b}}{8x^2}-\frac{e^{2a}b\,\mathrm{expIntegral}(1,-2x^2b)}{4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^2-1/8*exp(-2*a)/x^2*exp(-2*x^2*b)+1/4*exp(-2*a)*b*Ei(1,2*x^2*b)-1/8*
exp(2*a)/x^2*exp(2*x^2*b)-1/4*exp(2*a)*b*Ei(1,-2*x^2*b)
```

Maxima [A]
time = 0.32, size = 36, normalized size = 0.63

$$-\frac{1}{4}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx^2\right)+\frac{1}{4}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx^2\right)-\frac{1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^(-2*a)*gamma(-1, 2*b*x^2) + 1/4*b*e^(2*a)*gamma(-1, -2*b*x^2) - 1/
4/x^2
```

Fricas [A]
time = 0.35, size = 90, normalized size = 1.58

$$-\frac{\cosh\left(bx^2+a\right)^2-\left(bx^2\mathrm{Ei}(2\,bx^2)-bx^2\mathrm{Ei}(-2\,bx^2)\right)\cosh\left(2\,a\right)+\sinh\left(bx^2+a\right)^2-\left(bx^2\mathrm{Ei}(2\,bx^2)+bx^2\mathrm{Ei}(-2\,bx^2)\right)\sinh\left(2\,a\right)+1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(cosh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) - b*x^2*Ei(-2*b*x^2))*cosh(2*a
) + sinh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) + b*x^2*Ei(-2*b*x^2))*sinh(2*a)
+ 1)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^2\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)**2/x**3,x)
```

```
[Out] Integral(cosh(a + b*x**2)**2/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(50) = 100$.
time = 0.41, size = 126, normalized size = 2.21

$$\frac{2\,(bx^2+a)b^2\mathrm{Ei}(2\,bx^2)\,e^{(2\,a)} - 2\,ab^2\mathrm{Ei}(2\,bx^2)\,e^{(2\,a)} - 2\,(bx^2+a)b^2\mathrm{Ei}(-2\,bx^2)\,e^{(-2\,a)} + 2\,ab^2\mathrm{Ei}(-2\,bx^2)\,e^{(-2\,a)} - b^2e^{(2\,bx^2+2\,a)} - b^2e^{(-2\,bx^2-2\,a)} - 2\,b^2}{8\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^2 + a)*b^2*Ei(2*b*x^2)*e^(2*a) - 2*a*b^2*Ei(2*b*x^2)*e^(2*a) -
2*(b*x^2 + a)*b^2*Ei(-2*b*x^2)*e^(-2*a) + 2*a*b^2*Ei(-2*b*x^2)*e^(-2*a) - b
^2*e^(2*b*x^2 + 2*a) - b^2*e^(-2*b*x^2 - 2*a) - 2*b^2)/(b^2*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cosh\left(b\,x^2+a\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)^2/x^3,x)
```

```
[Out] int(cosh(a + b*x^2)^2/x^3, x)
```

3.15 $\int x^3 \cosh^3(a+bx^2)\,dx$

Optimal. Leaf size=79

$$-\frac{\cosh(a+bx^2)}{3b^2}-\frac{\cosh^3(a+bx^2)}{18b^2}+\frac{x^2\sinh(a+bx^2)}{3b}+\frac{x^2\cosh^2(a+bx^2)\sinh(a+bx^2)}{6b}$$

```
[Out] -1/3*cosh(b*x^2+a)/b^2-1/18*cosh(b*x^2+a)^3/b^2+1/3*x^2*sinh(b*x^2+a)/b+1/6
*x^2*cosh(b*x^2+a)^2*sinh(b*x^2+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {5429, 3391, 3377, 2718}

$$-\frac{\cosh^3(a+bx^2)}{18b^2}-\frac{\cosh(a+bx^2)}{3b^2}+\frac{x^2\sinh(a+bx^2)}{3b}+\frac{x^2\sinh(a+bx^2)\cosh^2(a+bx^2)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x^2]^3,x]
```

```
[Out] -1/3*Cosh[a + b*x^2]/b^2 - Cosh[a + b*x^2]^3/(18*b^2) + (x^2*Sinh[a + b*x^2
])/(3*b) + (x^2*Cosh[a + b*x^2]^2*Sinh[a + b*x^2])/(6*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
```

```
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \cosh^3\left(a+bx^2\right)\,dx = \frac{1}{2}\mathrm{Subst}\left(\int x\cosh^3(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{\cosh^3\left(a+bx^2\right)}{18b^2} + \frac{x^2\cosh^2\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{6b} + \frac{1}{3}\mathrm{Subst}\left(\int x\cosh(a+$$
$$= -\frac{\cosh^3\left(a+bx^2\right)}{18b^2} + \frac{x^2\sinh\left(a+bx^2\right)}{3b} + \frac{x^2\cosh^2\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{6b} - \mathrm{Sub}$$
$$= -\frac{\cosh\left(a+bx^2\right)}{3b^2} - \frac{\cosh^3\left(a+bx^2\right)}{18b^2} + \frac{x^2\sinh\left(a+bx^2\right)}{3b} + \frac{x^2\cosh^2\left(a+bx^2\right)\sinh}{6b}$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 0.70

$$-\frac{27\cosh\left(a+bx^2\right)+\cosh\left(3\left(a+bx^2\right)\right)-3bx^2\left(9\sinh\left(a+bx^2\right)+\sinh\left(3\left(a+bx^2\right)\right)\right)}{72b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x^2]^3,x]
```

```
[Out] -1/72*(27*Cosh[a + b*x^2] + Cosh[3*(a + b*x^2)] - 3*b*x^2*(9*Sinh[a + b*x^2
] + Sinh[3*(a + b*x^2)]))/b^2
```

Maple [A]
time = 1.86, size = 93, normalized size = 1.18

method	result	size
risch	$\frac{(3x^2b-1)\,\mathrm{e}^{3x^2b+3a}}{144b^2}+\frac{3(x^2b-1)\,\mathrm{e}^{x^2b+a}}{16b^2}-\frac{3(x^2b+1)\,\mathrm{e}^{-x^2b-a}}{16b^2}-\frac{(3x^2b+1)\,\mathrm{e}^{-3x^2b-3a}}{144b^2}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/144*(3*b*x^2-1)/b^2*exp(3*b*x^2+3*a)+3/16*(b*x^2-1)/b^2*exp(b*x^2+a)-3/16
*(b*x^2+1)/b^2*exp(-b*x^2-a)-1/144*(3*b*x^2+1)/b^2*exp(-3*b*x^2-3*a)
```

Maxima [A]
time = 0.26, size = 100, normalized size = 1.27

$$\frac{\left(3\,bx^2e^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx^2)}}{144\,b^2}+\frac{3\,\left(bx^2e^a-e^a\right)e^{(bx^2)}}{16\,b^2}-\frac{3\,\left(bx^2+1\right)e^{(-bx^2-a)}}{16\,b^2}-\frac{\left(3\,bx^2+1\right)e^{(-3\,bx^2-3\,a)}}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/144*(3*b*x^2*e^(3*a) - e^(3*a))*e^(3*b*x^2)/b^2 + 3/16*(b*x^2*e^a - e^a)*
e^(b*x^2)/b^2 - 3/16*(b*x^2 + 1)*e^(-b*x^2 - a)/b^2 - 1/144*(3*b*x^2 + 1)*e
^(-3*b*x^2 - 3*a)/b^2
```

Fricas [A]
time = 0.40, size = 96, normalized size = 1.22

$$\frac{3\,bx^2\sinh\left(bx^2+a\right)^3-\cosh\left(bx^2+a\right)^3-3\,\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2+9\left(bx^2\cosh\left(bx^2+a\right)^2+3\,bx^2\right)\sinh\left(bx^2+a\right)-27\,\cosh\left(bx^2+a\right)}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72*(3*b*x^2*sinh(b*x^2 + a)^3 - cosh(b*x^2 + a)^3 - 3*cosh(b*x^2 + a)*sin
h(b*x^2 + a)^2 + 9*(b*x^2*cosh(b*x^2 + a)^2 + 3*b*x^2)*sinh(b*x^2 + a) - 27
*cosh(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.43, size = 92, normalized size = 1.16

$$\begin{cases} -\frac{x^2\sinh^3\left(a+bx^2\right)}{3b}+\frac{x^2\sinh\left(a+bx^2\right)\cosh^2\left(a+bx^2\right)}{2b}+\frac{\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{3b^2}-\frac{7\cosh^3\left(a+bx^2\right)}{18b^2} & \text{for } b\neq 0 \\ \frac{x^4\cosh^3\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x**2+a)**3,x)
```

```
[Out] Piecewise((-x**2*sinh(a + b*x**2)**3/(3*b) + x**2*sinh(a + b*x**2)*cosh(a +
 b*x**2)**2/(2*b) + sinh(a + b*x**2)**2*cosh(a + b*x**2)/(3*b**2) - 7*cosh(
a + b*x**2)**3/(18*b**2), Ne(b, 0)), (x**4*cosh(a)**3/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(71) = 142.
time = 0.41, size = 192, normalized size = 2.43

$$\frac{3\,(bx^2+a)e^{(3\,bx^2+3\,a)}+27\,(bx^2+a)e^{(bx^2+a)}-27\,(bx^2+a)e^{(-bx^2-a)}-3\,(bx^2+a)e^{(-3\,bx^2-3\,a)}-e^{(3\,bx^2+3\,a)}-27\,e^{(bx^2+a)}-27\,e^{(-bx^2-a)}-e^{(-3\,bx^2-3\,a)}}{144\,b^2}-\frac{ae^{(3\,bx^2+3\,a)}+9\,ae^{(bx^2+a)}-\left(9\,ae^{(2\,bx^2+2\,a)}+a\right)e^{(-3\,bx^2-3\,a)}}{48\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/144*(3*(b*x^2 + a)*e^(3*b*x^2 + 3*a) + 27*(b*x^2 + a)*e^(b*x^2 + a) - 27*
(b*x^2 + a)*e^(-b*x^2 - a) - 3*(b*x^2 + a)*e^(-3*b*x^2 - 3*a) - e^(3*b*x^2
+ 3*a) - 27*e^(b*x^2 + a) - 27*e^(-b*x^2 - a) - e^(-3*b*x^2 - 3*a))/b^2 - 1
```

```
/48*(a*e^(3*b*x^2 + 3*a) + 9*a*e^(b*x^2 + a) - (9*a*e^(2*b*x^2 + 2*a) + a)*
e^(-3*b*x^2 - 3*a))/b^2
```

Mupad [B]
time = 0.96, size = 70, normalized size = 0.89

$$\frac{\frac{x^2\sinh\left(b\,x^2+a\right)}{3}+\frac{x^2\cosh\left(b\,x^2+a\right)^2\sinh\left(b\,x^2+a\right)}{6}}{b}-\frac{\cosh\left(b\,x^2+a\right)^3}{18\,b^2}-\frac{\cosh\left(b\,x^2+a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x^2)^3,x)
```

```
[Out] ((x^2*sinh(a + b*x^2))/3 + (x^2*cosh(a + b*x^2)^2*sinh(a + b*x^2))/6)/b - c
osh(a + b*x^2)^3/(18*b^2) - cosh(a + b*x^2)/(3*b^2)
```

3.16 $\int x^2 \cosh^3\left(a + bx^2\right)\, dx$

Optimal. Leaf size=160

$$\frac{3e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)}{32b^{3/2}}+\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{96b^{3/2}}-\frac{3e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{e^{3a}\sqrt{\frac{\pi}{3}}\,\mathrm{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{96b^{3/2}}+\frac{3x\sinh(a}{8b}$$

```
[Out] 3/8*x*sinh(b*x^2+a)/b+1/24*x*sinh(3*b*x^2+3*a)/b+1/288*erf(x*3^(1/2)*b^(1/2
))*3^(1/2)*Pi^(1/2)/b^(3/2)/exp(3*a)-1/288*exp(3*a)*erfi(x*3^(1/2)*b^(1/2))
*3^(1/2)*Pi^(1/2)/b^(3/2)+3/32*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)-3/32*
exp(a)*erfi(x*b^(1/2))*Pi^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5449, 5433, 5406, 2235, 2236}

$$\frac{3\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\sqrt{b}\,x\right)}{32b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\,e^{-3a}\mathrm{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{96b^{3/2}}-\frac{3\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\,e^{3a}\mathrm{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{96b^{3/2}}+\frac{3x\sinh\left(a+bx^2\right)}{8b}+\frac{x\sinh\left(3a+3bx^2\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x^2]^3,x]
```

```
[Out] (3*Sqrt[Pi]*Erf[Sqrt[b]*x])/(32*b^(3/2)*E^a) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt
[b]*x])/(96*b^(3/2)*E^(3*a)) - (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(32*b^(3/2)
) - (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(96*b^(3/2)) + (3*x*Sinh[a
 + b*x^2])/(8*b) + (x*Sinh[3*a + 3*b*x^2])/(24*b)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\cosh^3\left(a+bx^2\right)\,dx &= \int\left(\frac{3}{4}x^2\cosh\left(a+bx^2\right)+\frac{1}{4}x^2\cosh\left(3a+3bx^2\right)\right)dx\\
&= \frac{1}{4}\int x^2\cosh\left(3a+3bx^2\right)\,dx+\frac{3}{4}\int x^2\cosh\left(a+bx^2\right)\,dx\\
&= \frac{3x\sinh\left(a+bx^2\right)}{8b}+\frac{x\sinh\left(3a+3bx^2\right)}{24b}-\frac{\int\sinh\left(3a+3bx^2\right)\,dx}{24b}-\frac{3\int\sinh\left(a+\right.}{8b}\\
&= \frac{3x\sinh\left(a+bx^2\right)}{8b}+\frac{x\sinh\left(3a+3bx^2\right)}{24b}+\frac{\int e^{-3a-3bx^2}\,dx}{48b}-\frac{\int e^{3a+3bx^2}\,dx}{48b}+\frac{3\int e}{}\\
&= \frac{3e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)}{32b^{3/2}}+\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}\,x\right)}{96b^{3/2}}-\frac{3e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{e^{3a}\sqrt{}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 184, normalized size = 1.15

$$\frac{-27\sqrt{\pi}\cosh(a)\mathrm{Erfi}\left(\sqrt{b}\,x\right)-\sqrt{3\pi}\cosh(3a)\mathrm{Erfi}\left(\sqrt{3}\sqrt{b}\,x\right)+27\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)(\cosh(a)-\sinh(a))-27\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)\sinh(a)+\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\sqrt{b}\,x\right)(\cosh(3a)-\sinh(3a))-\sqrt{3\pi}\,\mathrm{Erfi}\left(\sqrt{3}\sqrt{b}\,x\right)\sinh(3a)+108\sqrt{b}\,x\sinh\left(a+bx^2\right)+12\sqrt{b}\,x\sinh\left(3\left(a+bx^2\right)\right)}{288b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x^2]^3,x]
```

```
[Out] (-27*Sqrt[Pi]*Cosh[a]*Erfi[Sqrt[b]*x] - Sqrt[3*Pi]*Cosh[3*a]*Erfi[Sqrt[3]*S
qrt[b]*x] + 27*Sqrt[Pi]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) - 27*Sqrt[Pi]*Er
fi[Sqrt[b]*x]*Sinh[a] + Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[b]*x]*(Cosh[3*a] - Sinh
[3*a]) - Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + 108*Sqrt[b]*x*Sinh[
a + b*x^2] + 12*Sqrt[b]*x*Sinh[3*(a + b*x^2)])/(288*b^(3/2))
```

Maple [A]
time = 2.04, size = 157, normalized size = 0.98

method	result
risch	$-\frac{\mathrm{e}^{-3a} x\,\mathrm{e}^{-3x^2 b}}{48b}+\frac{\mathrm{e}^{-3a}\sqrt{\pi}\,\sqrt{3}\,\operatorname{erf}\left(x\sqrt{3}\,\sqrt{b}\right)}{288b^{\frac{3}{2}}}-\frac{3\,\mathrm{e}^{-a} x\,\mathrm{e}^{-x^2 b}}{16b}+\frac{3\,\mathrm{e}^{-a}\sqrt{\pi}\,\operatorname{erf}\left(x\sqrt{b}\right)}{32b^{\frac{3}{2}}}+\frac{3\,\mathrm{e}^{a}\mathrm{e}^{x^2 b} x}{16b}-\frac{3\,\mathrm{e}^{a}\sqrt{\pi}}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*exp(-3*a)/b*x*exp(-3*x^2*b)+1/288*exp(-3*a)/b^(3/2)*Pi^(1/2)*3^(1/2)*
erf(x*3^(1/2)*b^(1/2))-3/16*exp(-a)/b*x*exp(-x^2*b)+3/32*exp(-a)/b^(3/2)*Pi
^(1/2)*erf(x*b^(1/2))+3/16*exp(a)*exp(x^2*b)*x/b-3/32*exp(a)/b*Pi^(1/2)/(-b
)^(1/2)*erf((-b)^(1/2)*x)+1/48*exp(3*a)/b*x*exp(3*x^2*b)-1/96*exp(3*a)/b*Pi
^(1/2)/(-3*b)^(1/2)*erf((-3*b)^(1/2)*x)
```

Maxima [A]
time = 0.50, size = 162, normalized size = 1.01

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{3}\,\sqrt{-b}\,x\right)e^{(3\,a)}}{288\,\sqrt{-b}\,b}+\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)e^{(-3\,a)}}{288\,b^{\frac{3}{2}}}+\frac{xe^{(3\,bx^2+3\,a)}}{48\,b}+\frac{3\,xe^{(bx^2+a)}}{16\,b}-\frac{3\,xe^{(-bx^2-a)}}{16\,b}-\frac{xe^{(-3\,bx^2-3\,a)}}{48\,b}+\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,x\right)e^{(-a)}}{32\,b^{\frac{3}{2}}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)e^{a}}{32\,\sqrt{-b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/288*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) + 1/28
8*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3*b*
x^2 + 3*a)/b + 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b - 1/48*x*e^
(-3*b*x^2 - 3*a)/b + 3/32*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^(3/2) - 3/32*sqr
t(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(114) = 228$.
time = 0.40, size = 903, normalized size = 5.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/288*(6*b*x*cosh(b*x^2 + a)^6 + 36*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 +
 6*b*x*sinh(b*x^2 + a)^6 + 54*b*x*cosh(b*x^2 + a)^4 + 18*(5*b*x*cosh(b*x^2
+ a)^2 + 3*b*x)*sinh(b*x^2 + a)^4 - 54*b*x*cosh(b*x^2 + a)^2 + 24*(5*b*x*co
sh(b*x^2 + a)^3 + 9*b*x*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)*sqrt(p
i)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) + sinh(3*a))*sinh(b*x^2 + a)^3
 + cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a) + cosh(b*x^2
+ a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(3*a) + cosh(b
```

```
*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(3)*sqrt(-b)*x) +
sqrt(3)*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) - sinh(3*a))*sin
h(b*x^2 + a)^3 - cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a)
 - cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh
(3*a) - cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*s
qrt(b)*x) + 27*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) + sinh(a))*si
nh(b*x^2 + a)^3 + cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) +
cosh(b*x^2 + a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) +
 cosh(b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 27*
sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a)^3
 - cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) - cosh(b*x^2 + a)
*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) - cosh(b*x^2 + a
)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 18*(5*b*x*cosh(b*x^2
 + a)^4 + 18*b*x*cosh(b*x^2 + a)^2 - 3*b*x)*sinh(b*x^2 + a)^2 - 6*b*x + 36*
(b*x*cosh(b*x^2 + a)^5 + 6*b*x*cosh(b*x^2 + a)^3 - 3*b*x*cosh(b*x^2 + a))*s
inh(b*x^2 + a))/(b^2*cosh(b*x^2 + a)^3 + 3*b^2*cosh(b*x^2 + a)^2*sinh(b*x^2
 + a) + 3*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 + b^2*sinh(b*x^2 + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cosh^3\left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x**2+a)**3,x)
```

```
[Out] Integral(x**2*cosh(a + b*x**2)**3, x)
```

Giac [A]
time = 0.41, size = 166, normalized size = 1.04

$$\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{3}\,\sqrt{-b}\,x\right)e^{(3\,a)}}{288\,\sqrt{-b}\,b} - \frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{3}\,\sqrt{b}\,x\right)e^{(-3\,a)}}{288\,b^{\frac{3}{2}}} + \frac{xe^{(3\,bx^2+3\,a)}}{48\,b} + \frac{3\,xe^{(bx^2+a)}}{16\,b} - \frac{3\,xe^{(-bx^2-a)}}{16\,b} - \frac{xe^{(-3\,bx^2-3\,a)}}{48\,b} - \frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{b}\,x\right)e^{(-a)}}{32\,b^{\frac{3}{2}}} + \frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-b}\,x\right)e^{a}}{32\,\sqrt{-b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/288*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) - 1/28
8*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3*b
*x^2 + 3*a)/b + 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b - 1/48*x*e
^(-3*b*x^2 - 3*a)/b - 3/32*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/b^(3/2) + 3/32*s
qrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\cosh\left(b\,x^2+a\right)^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x^2)^3,x)
[Out] int(x^2*cosh(a + b*x^2)^3, x)
```

3.17 $\int x\cosh^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=33

$$\frac{\sinh\left(a+bx^2\right)}{2b}+\frac{\sinh^3\left(a+bx^2\right)}{6b}$$

```
[Out] 1/2*sinh(b*x^2+a)/b+1/6*sinh(b*x^2+a)^3/b
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {5429, 2713}

$$\frac{\sinh^3\left(a+bx^2\right)}{6b}+\frac{\sinh\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x^2]^3,x]
```

```
[Out] Sinh[a + b*x^2]/(2*b) + Sinh[a + b*x^2]^3/(6*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\cosh^3\left(a+bx^2\right)\,dx&=\frac{1}{2}\mathrm{Subst}\left(\int\cosh^3(a+bx)\,dx,x,x^2\right)\\&=\frac{i\mathrm{Subst}\left(\int\left(1-x^2\right)\,dx,x,-i\sinh\left(a+bx^2\right)\right)}{2b}\\&=\frac{\sinh\left(a+bx^2\right)}{2b}+\frac{\sinh^3\left(a+bx^2\right)}{6b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{\sinh(a+bx^2)}{2b}+\frac{\sinh^3(a+bx^2)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x^2]^3,x]
```

```
[Out] Sinh[a + b*x^2]/(2*b) + Sinh[a + b*x^2]^3/(6*b)
```

Maple [A]
time = 1.07, size = 31, normalized size = 0.94

method	result	size
default	$\frac{3\sinh(x^2b+a)}{8b}+\frac{\sinh(3x^2b+3a)}{24b}$	31
risch	$\frac{e^{3x^2b+3a}}{48b}+\frac{3\,e^{x^2b+a}}{16b}-\frac{3\,e^{-x^2b-a}}{16b}-\frac{e^{-3x^2b-3a}}{48b}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*sinh(b*x^2+a)/b+1/24/b*sinh(3*b*x^2+3*a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.
time = 0.26, size = 62, normalized size = 1.88

$$\frac{e^{(3\,bx^2+3\,a)}}{48\,b}+\frac{3\,e^{(bx^2+a)}}{16\,b}-\frac{3\,e^{(-bx^2-a)}}{16\,b}-\frac{e^{(-3\,bx^2-3\,a)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/48*e^(3*b*x^2 + 3*a)/b + 3/16*e^(b*x^2 + a)/b - 3/16*e^(-b*x^2 - a)/b - 1
/48*e^(-3*b*x^2 - 3*a)/b
```

Fricas [A]
time = 0.36, size = 38, normalized size = 1.15

$$\frac{\sinh(bx^2+a)^3+3\left(\cosh(bx^2+a)^2+3\right)\sinh(bx^2+a)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(sinh(b*x^2 + a)^3 + 3*(cosh(b*x^2 + a)^2 + 3)*sinh(b*x^2 + a))/b
```

Sympy [A]
time = 0.19, size = 44, normalized size = 1.33

$$\begin{cases} -\frac{\sinh^3(a+bx^2)}{3b} + \frac{\sinh(a+bx^2)\cosh^2(a+bx^2)}{2b} & \text{for } b \neq 0 \\ \frac{x^2\cosh^3(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x**2+a)**3,x)
[Out] Piecewise((-sinh(a + b*x**2)**3/(3*b) + sinh(a + b*x**2)*cosh(a + b*x**2)**
2/(2*b), Ne(b, 0)), (x**2*cosh(a)**3/2, True))
```

Giac [A]
time = 0.42, size = 56, normalized size = 1.70

$$-\frac{\left(9\,e^{\left(2\,bx^2+2\,a\right)}+1\right)e^{\left(-3\,bx^2-3\,a\right)}-e^{\left(3\,bx^2+3\,a\right)}-9\,e^{\left(bx^2+a\right)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a)^3,x, algorithm="giac")
[Out] -1/48*((9*e^(2*b*x^2 + 2*a) + 1)*e^(-3*b*x^2 - 3*a) - e^(3*b*x^2 + 3*a) - 9
*e^(b*x^2 + a))/b
```

Mupad [B]
time = 0.06, size = 26, normalized size = 0.79

$$\frac{\sinh(b\,x^2+a)^3+3\sinh(b\,x^2+a)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x^2)^3,x)
[Out] (3*sinh(a + b*x^2) + sinh(a + b*x^2)^3)/(6*b)
```

3.18 $\int \cosh^3(a+bx^2)\,dx$

Optimal. Leaf size=125

$$\frac{3e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{3e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{e^{3a}\sqrt{\frac{\pi}{3}}\,\mathrm{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}$$

```
[Out] 1/48*erf(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/exp(3*a)/b^(1/2)+1/48*exp(3*a)
*erfi(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/b^(1/2)+3/16*erf(x*b^(1/2))*Pi^(1
/2)/exp(a)/b^(1/2)+3/16*exp(a)*erfi(x*b^(1/2))*Pi^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5409, 5407, 2235, 2236}

$$\frac{3\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}\,e^{-3a}\mathrm{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{3\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}\,e^{3a}\mathrm{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]^3,x]
```

```
[Out] (3*Sqrt[Pi]*Erf[Sqrt[b]*x])/(16*Sqrt[b]*E^a) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt
[b]*x])/(16*Sqrt[b]*E^(3*a)) + (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(16*Sqrt[b]
) + (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(16*Sqrt[b])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5409

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\int \cosh^3\left(a+bx^2\right)\,dx &= \int \left(\frac{3}{4}\cosh\left(a+bx^2\right)+\frac{1}{4}\cosh\left(3a+3bx^2\right)\right)dx\\
&= \frac{1}{4}\int \cosh\left(3a+3bx^2\right)\,dx+\frac{3}{4}\int \cosh\left(a+bx^2\right)\,dx\\
&= \frac{1}{8}\int e^{-3a-3bx^2}\,dx+\frac{1}{8}\int e^{3a+3bx^2}\,dx+\frac{3}{8}\int e^{-a-bx^2}\,dx+\frac{3}{8}\int e^{a+bx^2}\,dx\\
&= \frac{3e^{-a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\operatorname{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{3e^{a}\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{e^{3a}\sqrt{\frac{\pi}{3}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 136, normalized size = 1.09

$$\frac{\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}\cosh(a)\text{Erfi}\left(\sqrt{b}\,x\right)+\cosh(3a)\text{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)+3\sqrt{3}\,\text{Erf}\left(\sqrt{b}\,x\right)(\cosh(a)-\sinh(a))+3\sqrt{3}\,\text{Erfi}\left(\sqrt{b}\,x\right)\sinh(a)+\text{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)(\cosh(3a)-\sinh(3a))+\text{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)\sinh(3a)\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]^3,x]
```

```
[Out] (Sqrt[Pi/3]*(3*Sqrt[3]*Cosh[a]*Erfi[Sqrt[b]*x] + Cosh[3*a]*Erfi[Sqrt[3]*Sqr
t[b]*x] + 3*Sqrt[3]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + 3*Sqrt[3]*Erfi[Sqr
t[b]*x]*Sinh[a] + Erf[Sqrt[3]*Sqrt[b]*x]*(Cosh[3*a] - Sinh[3*a]) + Erfi[Sqr
t[3]*Sqrt[b]*x]*Sinh[3*a]))/(16*Sqrt[b])
```

Maple [A]
time = 1.46, size = 86, normalized size = 0.69

method	result
risch	$\frac{\mathrm{e}^{-3a}\sqrt{\pi}\,\sqrt{3}\,\operatorname{erf}\left(x\sqrt{3}\,\sqrt{b}\right)}{48\sqrt{b}}+\frac{3\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\,\mathrm{e}^{-a}}{16\sqrt{b}}+\frac{\mathrm{e}^{3a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-3b}\,x\right)}{16\sqrt{-3b}}+\frac{3\,\mathrm{e}^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)}{16\sqrt{-b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*exp(-3*a)*Pi^(1/2)*3^(1/2)/b^(1/2)*erf(x*3^(1/2)*b^(1/2))+3/16*erf(x*b
^(1/2))*Pi^(1/2)*exp(-a)/b^(1/2)+1/16*exp(3*a)*Pi^(1/2)/(-3*b)^(1/2)*erf((-
3*b)^(1/2)*x)+3/16*exp(a)*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Maxima [A]
time = 0.48, size = 91, normalized size = 0.73

$$\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{-b}\,x\right)e^{(3\,a)}}{48\sqrt{-b}}+\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}\,x\right)e^{(-3\,a)}}{48\sqrt{b}}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)e^{(-a)}}{16\sqrt{b}}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b}\,x\right)e^{a}}{16\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) + 1/48*sqrt(
3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) + 3/16*sqrt(pi)*erf(sqr
t(b)*x)*e^(-a)/sqrt(b) + 3/16*sqrt(pi)*erf(sqrt(-b)*x)*e^a/sqrt(-b)
```

Fricas [A]
time = 0.42, size = 113, normalized size = 0.90

$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{-b}\,(\cosh(3\,a)+\sinh(3\,a))\,\mathrm{erf}\left(\sqrt{3}\sqrt{-b}\,x\right)-\sqrt{3}\sqrt{\pi}\sqrt{b}\,(\cosh(3\,a)-\sinh(3\,a))\,\mathrm{erf}\left(\sqrt{3}\sqrt{b}\,x\right)+9\sqrt{\pi}\sqrt{-b}\,(\cosh(a)+\sinh(a))\,\mathrm{erf}\left(\sqrt{-b}\,x\right)-9\sqrt{\pi}\sqrt{b}\,(\cosh(a)-\sinh(a))\,\mathrm{erf}\left(\sqrt{b}\,x\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-b)*(cosh(3*a) + sinh(3*a))*erf(sqrt(3)*sqrt(-
b)*x) - sqrt(3)*sqrt(pi)*sqrt(b)*(cosh(3*a) - sinh(3*a))*erf(sqrt(3)*sqrt(b
)*x) + 9*sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) - 9*sqrt(pi)
*sqrt(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh^3\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)**3,x)
```

```
[Out] Integral(cosh(a + b*x**2)**3, x)
```

Giac [A]
time = 0.42, size = 95, normalized size = 0.76

$$-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{-b}\,x\right)e^{(3\,a)}}{48\sqrt{-b}}-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{3}\sqrt{b}\,x\right)e^{(-3\,a)}}{48\sqrt{b}}-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}\,x\right)e^{(-a)}}{16\sqrt{b}}-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}\,x\right)e^{a}}{16\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) - 1/48*sqr
t(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) - 3/16*sqrt(pi)*erf(
-sqrt(b)*x)*e^(-a)/sqrt(b) - 3/16*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/sqrt(-b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh \left(b\,x^2+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)^3,x)
```

```
[Out] int(cosh(a + b*x^2)^3, x)
```

3.19 $\int \frac{\cosh^3(a+bx^2)}{x} dx$

Optimal. Leaf size=55

$$\frac{3}{8}\cosh(a)\text{Chi}(bx^2)+\frac{1}{8}\cosh(3a)\text{Chi}(3bx^2)+\frac{3}{8}\sinh(a)\text{Shi}(bx^2)+\frac{1}{8}\sinh(3a)\text{Shi}(3bx^2)$$

```
[Out] 3/8*Chi(b*x^2)*cosh(a)+1/8*Chi(3*b*x^2)*cosh(3*a)+3/8*Shi(b*x^2)*sinh(a)+1/
8*Shi(3*b*x^2)*sinh(3*a)
```

Rubi [A]
time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5449, 5427, 5425, 5424}

$$\frac{3}{8}\cosh(a)\text{Chi}(bx^2)+\frac{1}{8}\cosh(3a)\text{Chi}(3bx^2)+\frac{3}{8}\sinh(a)\text{Shi}(bx^2)+\frac{1}{8}\sinh(3a)\text{Shi}(3bx^2)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]^3/x,x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x^2])/8 + (Cosh[3*a]*CoshIntegral[3*b*x^2])/8 + (
3*Sinh[a]*SinhIntegral[b*x^2])/8 + (Sinh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx^2)}{x}\,dx &= \int \left(\frac{3\cosh(a+bx^2)}{4x}+\frac{\cosh(3a+3bx^2)}{4x}\right)dx\\ &= \frac{1}{4}\int \frac{\cosh(3a+3bx^2)}{x}\,dx+\frac{3}{4}\int \frac{\cosh(a+bx^2)}{x}\,dx\\ &= \frac{1}{4}(3\cosh(a))\int \frac{\cosh(bx^2)}{x}\,dx+\frac{1}{4}\cosh(3a)\int \frac{\cosh(3bx^2)}{x}\,dx+\frac{1}{4}(3\sinh(a))\int \frac{\mathrm{sinl}}{}\\ &= \frac{3}{8}\cosh(a)\mathrm{Chi}\left(bx^2\right)+\frac{1}{8}\cosh(3a)\mathrm{Chi}\left(3bx^2\right)+\frac{3}{8}\sinh(a)\mathrm{Shi}\left(bx^2\right)+\frac{1}{8}\sinh(3a)\mathrm{Shi}(3\end{aligned}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.89

$$\frac{1}{8}\left(3\cosh(a)\mathrm{Chi}\left(bx^2\right)+\cosh(3a)\mathrm{Chi}\left(3bx^2\right)+3\sinh(a)\mathrm{Shi}\left(bx^2\right)+\sinh(3a)\mathrm{Shi}\left(3bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]^3/x,x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x^2] + Cosh[3*a]*CoshIntegral[3*b*x^2] + 3*Sinh[a
]*SinhIntegral[b*x^2] + Sinh[3*a]*SinhIntegral[3*b*x^2])/8
```

Maple [A]
time = 1.63, size = 55, normalized size = 1.00

method	result	size
risch	$-\frac{e^{-3a}\,\mathrm{expIntegral}\left(1,3x^2b\right)}{16}-\frac{3\,e^{-a}\,\mathrm{expIntegral}\left(1,x^2b\right)}{16}-\frac{3\,e^{a}\,\mathrm{expIntegral}\left(1,-x^2b\right)}{16}-\frac{e^{3a}\,\mathrm{expIntegral}\left(1,-3x^2b\right)}{16}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*exp(-3*a)*Ei(1,3*x^2*b)-3/16*exp(-a)*Ei(1,x^2*b)-3/16*exp(a)*Ei(1,-x^
2*b)-1/16*exp(3*a)*Ei(1,-3*x^2*b)
```

Maxima [A]
time = 0.33, size = 50, normalized size = 0.91

$$\frac{1}{16}\,\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)}+\frac{3}{16}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+\frac{1}{16}\,\mathrm{Ei}\left(-3\,bx^2\right)e^{(-3\,a)}+\frac{3}{16}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) + 1/16*Ei(-3*b*x^2)*e^(-3
*a) + 3/16*Ei(b*x^2)*e^a
```

Fricas [A]
time = 0.34, size = 83, normalized size = 1.51

$$\frac{1}{16}\left(\mathrm{Ei}\left(3\,bx^2\right)+\mathrm{Ei}\left(-3\,bx^2\right)\right)\cosh\left(3\,a\right)+\frac{3}{16}\left(\mathrm{Ei}\left(bx^2\right)+\mathrm{Ei}\left(-bx^2\right)\right)\cosh\left(a\right)+\frac{1}{16}\left(\mathrm{Ei}\left(3\,bx^2\right)-\mathrm{Ei}\left(-3\,bx^2\right)\right)\sinh\left(3\,a\right)+\frac{3}{16}\left(\mathrm{Ei}\left(bx^2\right)-\mathrm{Ei}\left(-bx^2\right)\right)\sinh\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(3*b*x^2) + Ei(-3*b*x^2))*cosh(3*a) + 3/16*(Ei(b*x^2) + Ei(-b*x^2))
*cosh(a) + 1/16*(Ei(3*b*x^2) - Ei(-3*b*x^2))*sinh(3*a) + 3/16*(Ei(b*x^2) -
Ei(-b*x^2))*sinh(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^3\left(a+bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)**3/x,x)
```

```
[Out] Integral(cosh(a + b*x**2)**3/x, x)
```

Giac [A]
time = 0.41, size = 50, normalized size = 0.91

$$\frac{1}{16}\,\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)}+\frac{3}{16}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+\frac{1}{16}\,\mathrm{Ei}\left(-3\,bx^2\right)e^{(-3\,a)}+\frac{3}{16}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) + 1/16*Ei(-3*b*x^2)*e^(-3
*a) + 3/16*Ei(b*x^2)*e^a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\mathrm{cosh}\left(b\,x^2+a\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)^3/x,x)
```

```
[Out] int(cosh(a + b*x^2)^3/x, x)
```

3.20 $\int \frac{\cosh^3(a+bx^2)}{x^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\cosh^3(a+bx^2)}{x}-\frac{3}{8}\sqrt{b}\,e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)-\frac{1}{8}\sqrt{b}\,e^{-3a}\sqrt{3\pi}\,\mathrm{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)+\frac{3}{8}\sqrt{b}\,e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)+\frac{1}{8}$$

```
[Out] -cosh(b*x^2+a)^3/x-3/8*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)+3/8*exp(a)*er
fi(x*b^(1/2))*b^(1/2)*Pi^(1/2)-1/8*erf(x*3^(1/2)*b^(1/2))*b^(1/2)*3^(1/2)*P
i^(1/2)/exp(3*a)+1/8*exp(3*a)*erfi(x*3^(1/2)*b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1
/2)
```

Rubi [A]
time = 0.08, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5439, 5737, 5406, 2235, 2236}

$$-\frac{3}{8}\sqrt{\pi}\,e^{-a}\sqrt{b}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)-\frac{1}{8}\sqrt{3\pi}\,e^{-3a}\sqrt{b}\,\mathrm{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)+\frac{3}{8}\sqrt{\pi}\,e^{a}\sqrt{b}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)+\frac{1}{8}\sqrt{3\pi}\,e^{3a}\sqrt{b}\,\mathrm{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)-\frac{\cosh^3(a+bx^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]^3/x^2,x]
```

```
[Out] -(Cosh[a + b*x^2]^3/x) - (3*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*E^a) - (Sqr
t[b]*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[b]*x])/(8*E^(3*a)) + (3*Sqrt[b]*E^a*Sqrt[P
i]*Erfi[Sqrt[b]*x])/8 + (Sqrt[b]*E^(3*a)*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x]
)/8
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5439

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[-Cosh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Dist[b*n*(p/(n - 1)), Int[Cosh[a + b
*x^n]^(p - 1)*Sinh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n, p
] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\cosh^3\left(a+bx^2\right)}{x}+(6b)\int \cosh^2\left(a+bx^2\right)\sinh\left(a+bx^2\right)\,dx\\
&= -\frac{\cosh^3\left(a+bx^2\right)}{x}+(6b)\int \left(\frac{1}{4}\sinh\left(a+bx^2\right)+\frac{1}{4}\sinh\left(3a+3bx^2\right)\right)dx\\
&= -\frac{\cosh^3\left(a+bx^2\right)}{x}+\frac{1}{2}(3b)\int \sinh\left(a+bx^2\right)\,dx+\frac{1}{2}(3b)\int \sinh\left(3a+3bx^2\right)\,dx\\
&= -\frac{\cosh^3\left(a+bx^2\right)}{x}-\frac{1}{4}(3b)\int e^{-3a-3bx^2}\,dx-\frac{1}{4}(3b)\int e^{-a-bx^2}\,dx+\frac{1}{4}(3b)\int e^{a+bx^2}\,dx\\
&= -\frac{\cosh^3\left(a+bx^2\right)}{x}-\frac{3}{8}\sqrt{b}\,e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)-\frac{1}{8}\sqrt{b}\,e^{-3a}\sqrt{3\pi}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)+\frac{3}{8}\sqrt{}
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 204, normalized size = 1.50

$$
\frac{-6\cosh\left(a+bx^2\right)-2\cosh\left(3\left(a+bx^2\right)\right)+3\sqrt{b}\,\sqrt{\pi}\,x\cosh(a)\mathrm{Erfi}\left(\sqrt{b}\,x\right)+\sqrt{b}\,\sqrt{3\pi}\,x\cosh(3a)\mathrm{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)+3\sqrt{b}\,\sqrt{\pi}\,x\mathrm{Erfi}\left(\sqrt{b}\,x\right)\sinh(a)+3\sqrt{b}\,\sqrt{\pi}\,x\mathrm{Erf}\left(\sqrt{b}\,x\right)\left(-\cosh(a)+\sinh(a)\right)+\sqrt{b}\,\sqrt{3\pi}\,x\mathrm{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)\sinh(3a)+\sqrt{b}\,\sqrt{3\pi}\,x\mathrm{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)\left(-\cosh(3a)+\sinh(3a)\right)}{8x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]^3/x^2,x]
```

```
[Out] (-6*Cosh[a + b*x^2] - 2*Cosh[3*(a + b*x^2)] + 3*Sqrt[b]*Sqrt[Pi]*x*Cosh[a]*
Erfi[Sqrt[b]*x] + Sqrt[b]*Sqrt[3*Pi]*x*Cosh[3*a]*Erfi[Sqrt[3]*Sqrt[b]*x] +
3*Sqrt[b]*Sqrt[Pi]*x*Erfi[Sqrt[b]*x]*Sinh[a] + 3*Sqrt[b]*Sqrt[Pi]*x*Erf[Sqr
t[b]*x]*(-Cosh[a] + Sinh[a]) + Sqrt[b]*Sqrt[3*Pi]*x*Erfi[Sqrt[3]*Sqrt[b]*x]
*Sinh[3*a] + Sqrt[b]*Sqrt[3*Pi]*x*Erf[Sqrt[3]*Sqrt[b]*x]*(-Cosh[3*a] + Sinh
[3*a]))/(8*x)
```

Maple [A]
time = 1.82, size = 149, normalized size = 1.10

method	result
risch	$-\frac{e^{-3a}e^{-3x^2b}}{8x}-\frac{e^{-3a}\sqrt{b}\,\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(x\sqrt{3}\,\sqrt{b}\right)}{8}-\frac{3\,e^{-a}e^{-x^2b}}{8x}-\frac{3\,e^{-a}\sqrt{b}\,\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{b}\right)}{8}-\frac{3\,e^{a}e^{x^2b}}{8x}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*exp(-3*a)/x*exp(-3*x^2*b)-1/8*exp(-3*a)*b^(1/2)*Pi^(1/2)*3^(1/2)*erf(x
*3^(1/2)*b^(1/2))-3/8*exp(-a)/x*exp(-x^2*b)-3/8*exp(-a)*b^(1/2)*Pi^(1/2)*er
f(x*b^(1/2))-3/8*exp(a)*exp(x^2*b)/x+3/8*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((
-b)^(1/2)*x)-1/8*exp(3*a)/x*exp(3*x^2*b)+3/8*exp(3*a)*b*Pi^(1/2)/(-3*b)^(1/
2)*erf((-3*b)^(1/2)*x)
```

Maxima [A]
time = 0.34, size = 102, normalized size = 0.75

$$-\frac{\sqrt{3}\,\sqrt{bx^2}\,e^{(-3a)}\Gamma\left(-\frac{1}{2},3\,bx^2\right)}{16\,x}-\frac{\sqrt{3}\,\sqrt{-bx^2}\,e^{(3a)}\Gamma\left(-\frac{1}{2},-3\,bx^2\right)}{16\,x}-\frac{3\,\sqrt{bx^2}\,e^{(-a)}\Gamma\left(-\frac{1}{2},bx^2\right)}{16\,x}-\frac{3\,\sqrt{-bx^2}\,e^{a}\Gamma\left(-\frac{1}{2},-bx^2\right)}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(3)*sqrt(b*x^2)*e^(-3*a)*gamma(-1/2, 3*b*x^2)/x - 1/16*sqrt(3)*sq
rt(-b*x^2)*e^(3*a)*gamma(-1/2, -3*b*x^2)/x - 3/16*sqrt(b*x^2)*e^(-a)*gamma(
-1/2, b*x^2)/x - 3/16*sqrt(-b*x^2)*e^a*gamma(-1/2, -b*x^2)/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(98) = 196$.
time = 0.44, size = 891, normalized size = 6.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(b*x^2 + a)^6 + 6*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 + sinh(b*x^2
+ a)^6 + 3*(5*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^4 + 3*cosh(b*x^2 + a)^
4 + 4*(5*cosh(b*x^2 + a)^3 + 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)
*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3*a) + x*cosh(b*x^2 + a)^3*sinh(3*a) +
(x*cosh(3*a) + x*sinh(3*a))*sinh(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(3
*a) + x*cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)
^2*cosh(3*a) + x*cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf
(sqrt(3)*sqrt(-b)*x) + sqrt(3)*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3*a) - x*
cosh(b*x^2 + a)^3*sinh(3*a) + (x*cosh(3*a) - x*sinh(3*a))*sinh(b*x^2 + a)^3
```

```
 + 3*(x*cosh(b*x^2 + a)*cosh(3*a) - x*cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2
 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(3*a) - x*cosh(b*x^2 + a)^2*sinh(3*a))
*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*sqrt(b)*x) + 3*sqrt(pi)*(x*cosh(b*x^2
 + a)^3*cosh(a) + x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a) + x*sinh(a))*sin
h(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2 + a)*sinh(a))*
sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) + x*cosh(b*x^2 + a)^2*si
nh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 3*sqrt(pi)*(x*cosh(b*x^2
 + a)^3*cosh(a) - x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a) - x*sinh(a))*sin
h(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*x^2 + a)*sinh(a))*
sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) - x*cosh(b*x^2 + a)^2*si
nh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 3*(5*cosh(b*x^2 + a)^4 + 6
*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^2 + 3*cosh(b*x^2 + a)^2 + 6*(cosh(b
*x^2 + a)^5 + 2*cosh(b*x^2 + a)^3 + cosh(b*x^2 + a))*sinh(b*x^2 + a) + 1)/(
x*cosh(b*x^2 + a)^3 + 3*x*cosh(b*x^2 + a)^2*sinh(b*x^2 + a) + 3*x*cosh(b*x^
2 + a)*sinh(b*x^2 + a)^2 + x*sinh(b*x^2 + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)**3/x**2,x)
[Out] Integral(cosh(a + b*x**2)**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3/x^2,x, algorithm="giac")
[Out] integrate(cosh(b*x^2 + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(b\,x^2+a\right)}^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)^3/x^2,x)
[Out] int(cosh(a + b*x^2)^3/x^2, x)
```

3.21 $\int \frac{\cosh^3(a+bx^2)}{x^3}\,dx$

Optimal. Leaf size=91

$$-\frac{3\cosh\left(a+bx^2\right)}{8x^2}-\frac{\cosh\left(3(a+bx^2)\right)}{8x^2}+\frac{3}{8}b\mathrm{Chi}\left(bx^2\right)\sinh(a)+\frac{3}{8}b\mathrm{Chi}\left(3bx^2\right)\sinh(3a)+\frac{3}{8}b\cosh(a)\mathrm{Shi}\left(bx^2\right)+\frac{3}{8}$$

```
[Out] -3/8*cosh(b*x^2+a)/x^2-1/8*cosh(3*b*x^2+3*a)/x^2+3/8*b*cosh(a)*Shi(b*x^2)+3
/8*b*cosh(3*a)*Shi(3*b*x^2)+3/8*b*Chi(b*x^2)*sinh(a)+3/8*b*Chi(3*b*x^2)*sin
h(3*a)
```

Rubi [A]

time = 0.15, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5449, 5429, 3378, 3384, 3379, 3382}

$$\frac{3}{8}b\sinh(a)\mathrm{Chi}\left(bx^2\right)+\frac{3}{8}b\sinh(3a)\mathrm{Chi}\left(3bx^2\right)+\frac{3}{8}b\cosh(a)\mathrm{Shi}\left(bx^2\right)+\frac{3}{8}b\cosh(3a)\mathrm{Shi}\left(3bx^2\right)-\frac{3\cosh\left(a+bx^2\right)}{8x^2}-\frac{\cosh\left(3(a+bx^2)\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^2]^3/x^3,x]
```

```
[Out] (-3*Cosh[a + b*x^2])/(8*x^2) - Cosh[3*(a + b*x^2)]/(8*x^2) + (3*b*CoshInteg
ral[b*x^2]*Sinh[a])/8 + (3*b*CoshIntegral[3*b*x^2]*Sinh[3*a])/8 + (3*b*Cosh
[a]*SinhIntegral[b*x^2])/8 + (3*b*Cosh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3\left(a+bx^2\right)}{x^3}\,dx &= \int \left(\frac{3\cosh\left(a+bx^2\right)}{4x^3}+\frac{\cosh\left(3a+3bx^2\right)}{4x^3}\right)dx\\
&= \frac{1}{4}\int \frac{\cosh\left(3a+3bx^2\right)}{x^3}\,dx+\frac{3}{4}\int \frac{\cosh\left(a+bx^2\right)}{x^3}\,dx\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{\cosh(3a+3bx)}{x^2}\,dx,x,x^2\right)+\frac{3}{8}\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{3\cosh\left(a+bx^2\right)}{8x^2}-\frac{\cosh\left(3\left(a+bx^2\right)\right)}{8x^2}+\frac{1}{8}(3b)\text{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx,x,x^2\right)+\\
&= -\frac{3\cosh\left(a+bx^2\right)}{8x^2}-\frac{\cosh\left(3\left(a+bx^2\right)\right)}{8x^2}+\frac{1}{8}(3b\cosh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx,x,x^2\right.\\
&= -\frac{3\cosh\left(a+bx^2\right)}{8x^2}-\frac{\cosh\left(3\left(a+bx^2\right)\right)}{8x^2}+\frac{3}{8}b\text{Chi}\left(bx^2\right)\sinh(a)+\frac{3}{8}b\text{Chi}\left(3bx^2\right)\sinh(3a
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 92, normalized size = 1.01

$$\frac{-3\cosh\left(a+bx^2\right)-\cosh\left(3\left(a+bx^2\right)\right)+3bx^2\text{Chi}\left(bx^2\right)\sinh(a)+3bx^2\text{Chi}\left(3bx^2\right)\sinh(3a)+3bx^2\cosh(a)\text{Shi}\left(bx^2\right)+3bx^2\cosh(3a)\text{Shi}\left(3bx^2\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^2]^3/x^3,x]
```

```
[Out] (-3*Cosh[a + b*x^2] - Cosh[3*(a + b*x^2)] + 3*b*x^2*CoshIntegral[b*x^2]*Sin
h[a] + 3*b*x^2*CoshIntegral[3*b*x^2]*Sinh[3*a] + 3*b*x^2*Cosh[a]*SinhIntegr
al[b*x^2] + 3*b*x^2*Cosh[3*a]*SinhIntegral[3*b*x^2])/(8*x^2)
```

Maple [A]
time = 1.55, size = 120, normalized size = 1.32

method	result
risch	$-\frac{e^{-3a}e^{-3x^2b}}{16x^2}+\frac{3\,e^{-3a}b\,\mathrm{expIntegral}(1,3x^2b)}{16}-\frac{3\,e^{-a}e^{-x^2b}}{16x^2}+\frac{3\,e^{-a}b\,\mathrm{expIntegral}(1,x^2b)}{16}-\frac{3\,e^{a}e^{x^2b}}{16x^2}-\frac{3\,e^{a}b\,\mathrm{expIntegral}(1,$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*exp(-3*a)/x^2*exp(-3*x^2*b)+3/16*exp(-3*a)*b*Ei(1,3*x^2*b)-3/16*exp(-
a)/x^2*exp(-x^2*b)+3/16*exp(-a)*b*Ei(1,x^2*b)-3/16*exp(a)*exp(x^2*b)/x^2-3/
16*exp(a)*b*Ei(1,-x^2*b)-1/16*exp(3*a)/x^2*exp(3*x^2*b)-3/16*exp(3*a)*b*Ei(
1,-3*x^2*b)
```

Maxima [A]
time = 0.34, size = 58, normalized size = 0.64

$$-\frac{3}{16}\,be^{(-3\,a)}\Gamma\left(-1,3\,bx^2\right)-\frac{3}{16}\,be^{(-a)}\Gamma\left(-1,bx^2\right)+\frac{3}{16}\,be^{a}\Gamma\left(-1,-bx^2\right)+\frac{3}{16}\,be^{(3\,a)}\Gamma\left(-1,-3\,bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/16*b*e^(-3*a)*gamma(-1, 3*b*x^2) - 3/16*b*e^(-a)*gamma(-1, b*x^2) + 3/16
*b*e^a*gamma(-1, -b*x^2) + 3/16*b*e^(3*a)*gamma(-1, -3*b*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(80) = 160.
time = 0.36, size = 168, normalized size = 1.85

$$-\frac{2\,\cosh\left(bx^2+a\right)^3+6\,\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2-3\left(bx^2\mathrm{Ei}(3\,bx^2)-bx^2\mathrm{Ei}(-3\,bx^2)\right)\cosh\left(3\,a\right)-3\left(bx^2\mathrm{Ei}(bx^2)-bx^2\mathrm{Ei}(-bx^2)\right)\cosh\left(a\right)-3\left(bx^2\mathrm{Ei}(3\,bx^2)+bx^2\mathrm{Ei}(-3\,bx^2)\right)\sinh\left(3\,a\right)-3\left(bx^2\mathrm{Ei}(bx^2)+bx^2\mathrm{Ei}(-bx^2)\right)\sinh\left(a\right)+6\,\cosh\left(bx^2+a\right)}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*cosh(b*x^2 + a)^3 + 6*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 - 3*(b*x^2
*Ei(3*b*x^2) - b*x^2*Ei(-3*b*x^2))*cosh(3*a) - 3*(b*x^2*Ei(b*x^2) - b*x^2*E
i(-b*x^2))*cosh(a) - 3*(b*x^2*Ei(3*b*x^2) + b*x^2*Ei(-3*b*x^2))*sinh(3*a) -
 3*(b*x^2*Ei(b*x^2) + b*x^2*Ei(-b*x^2))*sinh(a) + 6*cosh(b*x^2 + a))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x**2+a)**3/x**3,x)
```

```
[Out] Integral(cosh(a + b*x**2)**3/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(80) = 160$.
time = 0.41, size = 224, normalized size = 2.46

$$\frac{3\,(bx^2+a)b^2\mathrm{Ei}(3\,bx^2)\,e^{(3\,a)} - 3\,ab^2\mathrm{Ei}(3\,bx^2)\,e^{(3\,a)} - 3\,(bx^2+a)b^2\mathrm{Ei}(-bx^2)\,e^{(-a)} + 3\,ab^2\mathrm{Ei}(-bx^2)\,e^{(-a)} - 3\,(bx^2+a)b^2\mathrm{Ei}(-3\,bx^2)\,e^{(-3\,a)} + 3\,ab^2\mathrm{Ei}(-3\,bx^2)\,e^{(-3\,a)} + 3\,(bx^2+a)b^2\mathrm{Ei}(bx^2)\,e^{a} - 3\,ab^2\mathrm{Ei}(bx^2)\,e^{a} - b^2e^{(3\,bx^2+3\,a)} - 3\,b^2e^{(bx^2+a)} - 3\,b^2e^{(-bx^2-a)} - b^2e^{(-3\,bx^2-3\,a)}}{16\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*(b*x^2 + a)*b^2*Ei(3*b*x^2)*e^(3*a) - 3*a*b^2*Ei(3*b*x^2)*e^(3*a) -
 3*(b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) + 3*a*b^2*Ei(-b*x^2)*e^(-a) - 3*(b*x^2
 + a)*b^2*Ei(-3*b*x^2)*e^(-3*a) + 3*a*b^2*Ei(-3*b*x^2)*e^(-3*a) + 3*(b*x^2
+ a)*b^2*Ei(b*x^2)*e^a - 3*a*b^2*Ei(b*x^2)*e^a - b^2*e^(3*b*x^2 + 3*a) - 3*
b^2*e^(b*x^2 + a) - 3*b^2*e^(-b*x^2 - a) - b^2*e^(-3*b*x^2 - 3*a))/(b^2*x^2
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(b\,x^2+a\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^2)^3/x^3,x)
```

```
[Out] int(cosh(a + b*x^2)^3/x^3, x)
```

3.22 $\int x \cosh^7(a+bx^2)\,dx$

Optimal. Leaf size=67

$$\frac{\sinh(a+bx^2)}{2b}+\frac{\sinh^3(a+bx^2)}{2b}+\frac{3\sinh^5(a+bx^2)}{10b}+\frac{\sinh^7(a+bx^2)}{14b}$$

```
[Out] 1/2*sinh(b*x^2+a)/b+1/2*sinh(b*x^2+a)^3/b+3/10*sinh(b*x^2+a)^5/b+1/14*sinh(
b*x^2+a)^7/b
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5429, 2713}

$$\frac{\sinh^7(a+bx^2)}{14b}+\frac{3\sinh^5(a+bx^2)}{10b}+\frac{\sinh^3(a+bx^2)}{2b}+\frac{\sinh(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x^2]^7,x]
```

```
[Out] Sinh[a + b*x^2]/(2*b) + Sinh[a + b*x^2]^3/(2*b) + (3*Sinh[a + b*x^2]^5)/(10
*b) + Sinh[a + b*x^2]^7/(14*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x\cosh^7(a+bx^2)\,dx &= \frac{1}{2}\text{Subst}\left(\int \cosh^7(a+bx)\,dx,x,x^2\right)\\
&= \frac{i\text{Subst}\left(\int(1-3x^2+3x^4-x^6)\,dx,x,-i\sinh(a+bx^2)\right)}{2b}\\
&= \frac{\sinh(a+bx^2)}{2b}+\frac{\sinh^3(a+bx^2)}{2b}+\frac{3\sinh^5(a+bx^2)}{10b}+\frac{\sinh^7(a+bx^2)}{14b}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 1.00

$$\frac{35\sinh\left(a+bx^2\right)}{128b}+\frac{7\sinh\left(3\left(a+bx^2\right)\right)}{128b}+\frac{7\sinh\left(5\left(a+bx^2\right)\right)}{640b}+\frac{\sinh\left(7\left(a+bx^2\right)\right)}{896b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x^2]^7,x]
```

```
[Out] (35*Sinh[a + b*x^2])/(128*b) + (7*Sinh[3*(a + b*x^2)])/(128*b) + (7*Sinh[5*
(a + b*x^2)])/(640*b) + Sinh[7*(a + b*x^2)]/(896*b)
```

Maple [A]
time = 1.55, size = 63, normalized size = 0.94

method	result	size
default	$\frac{35\sinh\left(x^{2} b+a\right)}{128 b}+\frac{7\sinh\left(3 x^{2} b+3 a\right)}{128 b}+\frac{7\sinh\left(5 x^{2} b+5 a\right)}{640 b}+\frac{\sinh\left(7 x^{2} b+7 a\right)}{896 b}$	63
risch	$\frac{{\mathrm e}^{7 x^{2} b+7 a}}{1792 b}+\frac{7\,{\mathrm e}^{5 x^{2} b+5 a}}{1280 b}+\frac{7\,{\mathrm e}^{3 x^{2} b+3 a}}{256 b}+\frac{35\,{\mathrm e}^{x^{2} b+a}}{256 b}-\frac{35\,{\mathrm e}^{-x^{2} b-a}}{256 b}-\frac{7\,{\mathrm e}^{-3 x^{2} b-3 a}}{256 b}-\frac{7\,{\mathrm e}^{-5 x^{2} b-5 a}}{1280 b}-\frac{{\mathrm e}^{-7 x^{2} b-7 a}}{1792 b}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 35/128*sinh(b*x^2+a)/b+7/128/b*sinh(3*b*x^2+3*a)+7/640/b*sinh(5*b*x^2+5*a)+
1/896/b*sinh(7*b*x^2+7*a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(59) = 118$.
time = 0.26, size = 126, normalized size = 1.88

$$\frac{e^{\left(7\,bx^2+7\,a\right)}}{1792\,b}+\frac{7\,e^{\left(5\,bx^2+5\,a\right)}}{1280\,b}+\frac{7\,e^{\left(3\,bx^2+3\,a\right)}}{256\,b}+\frac{35\,e^{\left(bx^2+a\right)}}{256\,b}-\frac{35\,e^{\left(-bx^2-a\right)}}{256\,b}-\frac{7\,e^{\left(-3\,bx^2-3\,a\right)}}{256\,b}-\frac{7\,e^{\left(-5\,bx^2-5\,a\right)}}{1280\,b}-\frac{e^{\left(-7\,bx^2-7\,a\right)}}{1792\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/1792*e^(7*b*x^2 + 7*a)/b + 7/1280*e^(5*b*x^2 + 5*a)/b + 7/256*e^(3*b*x^2
+ 3*a)/b + 35/256*e^(b*x^2 + a)/b - 35/256*e^(-b*x^2 - a)/b - 7/256*e^(-3*b
*x^2 - 3*a)/b - 7/1280*e^(-5*b*x^2 - 5*a)/b - 1/1792*e^(-7*b*x^2 - 7*a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(59) = 118$.
time = 0.41, size = 128, normalized size = 1.91

$$\frac{5\,\sinh\left(bx^2+a\right)^7+7\,\left(15\,\cosh\left(bx^2+a\right)^2+7\right)\sinh\left(bx^2+a\right)^5+35\,\left(5\,\cosh\left(bx^2+a\right)^4+14\,\cosh\left(bx^2+a\right)^2+7\right)\sinh\left(bx^2+a\right)^3+35\,\left(\cosh\left(bx^2+a\right)^6+7\,\cosh\left(bx^2+a\right)^4+21\,\cosh\left(bx^2+a\right)^2+35\right)\sinh\left(bx^2+a\right)}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/4480*(5*sinh(b*x^2 + a)^7 + 7*(15*cosh(b*x^2 + a)^2 + 7)*sinh(b*x^2 + a)^
5 + 35*(5*cosh(b*x^2 + a)^4 + 14*cosh(b*x^2 + a)^2 + 7)*sinh(b*x^2 + a)^3 +
 35*(cosh(b*x^2 + a)^6 + 7*cosh(b*x^2 + a)^4 + 21*cosh(b*x^2 + a)^2 + 35)*s
inh(b*x^2 + a))/b
```

Sympy [A]
time = 0.95, size = 94, normalized size = 1.40

$$\begin{cases} -\frac{8\sinh^{7}\left(a+bx^{2}\right)}{35b}+\frac{4\sinh^{5}\left(a+bx^{2}\right)\cosh^{2}\left(a+bx^{2}\right)}{5b}-\frac{\sinh^{3}\left(a+bx^{2}\right)\cosh^{4}\left(a+bx^{2}\right)}{b}+\frac{\sinh\left(a+bx^{2}\right)\cosh^{6}\left(a+bx^{2}\right)}{2b} & \text{for } b\neq 0 \\ \frac{x^{2}\cosh^{7}\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x**2+a)**7,x)
```

```
[Out] Piecewise((-8*sinh(a + b*x**2)**7/(35*b) + 4*sinh(a + b*x**2)**5*cosh(a + b
*x**2)**2/(5*b) - sinh(a + b*x**2)**3*cosh(a + b*x**2)**4/b + sinh(a + b*x*
*2)*cosh(a + b*x**2)**6/(2*b), Ne(b, 0)), (x**2*cosh(a)**7/2, True))
```

Giac [A]
time = 0.42, size = 108, normalized size = 1.61

$$-\frac{\left(1225\,e^{\left(6\,bx^{2}+6\,a\right)}+245\,e^{\left(4\,bx^{2}+4\,a\right)}+49\,e^{\left(2\,bx^{2}+2\,a\right)}+5\right)e^{\left(-7\,bx^{2}-7\,a\right)}-5\,e^{\left(7\,bx^{2}+7\,a\right)}-49\,e^{\left(5\,bx^{2}+5\,a\right)}-245\,e^{\left(3\,bx^{2}+3\,a\right)}-1225\,e^{\left(bx^{2}+a\right)}}{8960\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] -1/8960*((1225*e^(6*b*x^2 + 6*a) + 245*e^(4*b*x^2 + 4*a) + 49*e^(2*b*x^2 +
2*a) + 5)*e^(-7*b*x^2 - 7*a) - 5*e^(7*b*x^2 + 7*a) - 49*e^(5*b*x^2 + 5*a) -
 245*e^(3*b*x^2 + 3*a) - 1225*e^(b*x^2 + a))/b
```

Mupad [B]
time = 0.99, size = 52, normalized size = 0.78

$$\frac{5\sinh(b\,x^2+a)^7+21\sinh(b\,x^2+a)^5+35\sinh(b\,x^2+a)^3+35\sinh(b\,x^2+a)}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x^2)^7,x)
```

```
[Out] (35*sinh(a + b*x^2) + 35*sinh(a + b*x^2)^3 + 21*sinh(a + b*x^2)^5 + 5*sinh(
a + b*x^2)^7)/(70*b)
```

3.23 $\int x^2 \cosh(x^3)\,dx$

Optimal. Leaf size=8

$$\frac{\sinh(x^3)}{3}$$

```
[Out] 1/3*sinh(x^3)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5429, 2717}

$$\frac{\sinh(x^3)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[x^3],x]
```

```
[Out] Sinh[x^3]/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^2 \cosh(x^3)\,dx &= \frac{1}{3}\text{Subst}\left(\int \cosh(x)\,dx, x, x^3\right)\\ &= \frac{\sinh(x^3)}{3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sinh(x^3)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[x^3],x]
```

```
[Out] Sinh[x^3]/3
```

Maple [A]
time = 0.46, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\sinh(x^3)}{3}$	7
default	$\frac{\sinh(x^3)}{3}$	7
meijerg	$\frac{\sinh(x^3)}{3}$	7
risch	$\frac{e^{x^3}}{6} - \frac{e^{-x^3}}{6}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sinh(x^3)
```

Maxima [A]
time = 0.27, size = 6, normalized size = 0.75

$$\frac{1}{3}\sinh\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sinh(x^3)
```

Fricas [A]
time = 0.38, size = 6, normalized size = 0.75

$$\frac{1}{3}\sinh\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x^3),x, algorithm="fricas")
```

```
[Out] 1/3*sinh(x^3)
```

Sympy [A]
time = 0.12, size = 5, normalized size = 0.62

$$\frac{\sinh\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(x**3),x)
```

```
[Out] sinh(x**3)/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.
time = 0.42, size = 15, normalized size = 1.88

$$-\frac{1}{6}e^{\left(-x^3\right)} + \frac{1}{6}e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x^3),x, algorithm="giac")
```

```
[Out] -1/6*e^(-x^3) + 1/6*e^(x^3)
```

Mupad [B]
time = 0.88, size = 6, normalized size = 0.75

$$\frac{\sinh\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x^3),x)
```

```
[Out] sinh(x^3)/3
```

3.24 $\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx$

Optimal. Leaf size=8

$$-\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

```
[Out] -1/5*sinh(1/x^5)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5429, 2717}

$$-\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x^(-5)]/x^6,x]
```

```
[Out] -1/5*Sinh[x^(-5)]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6}\,dx &= -\left(\frac{1}{5}\text{Subst}\left(\int \cosh(x)\,dx, x, \frac{1}{x^5}\right)\right)\\ &= -\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$-\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x^(-5)]/x^6,x]
```

```
[Out] -1/5*Sinh[x^(-5)]
```

Maple [A]
time = 0.43, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
default	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
meijerg	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
risch	$-\frac{e^{\frac{1}{x^5}}}{10}+\frac{e^{-\frac{1}{x^5}}}{10}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(1/x^5)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*sinh(1/x^5)
```

Maxima [A]
time = 0.27, size = 6, normalized size = 0.75

$$-\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(1/x^5)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*sinh(x^(-5))
```

Fricas [A]
time = 0.41, size = 6, normalized size = 0.75

$$-\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(1/x^5)/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*sinh(x^(-5))
```

Sympy [A]
time = 2.93, size = 8, normalized size = 1.00

$$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(1/x**5)/x**6,x)
```

```
[Out] -sinh(x**(-5))/5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.
time = 0.41, size = 15, normalized size = 1.88

$$\frac{1}{10}\,e^{\left(-\frac{1}{x^5}\right)} - \frac{1}{10}\,e^{\left(\frac{1}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(1/x^5)/x^6,x, algorithm="giac")
```

```
[Out] 1/10*e^(-1/x^5) - 1/10*e^(x^(-5))
```

Mupad [B]
time = 0.90, size = 15, normalized size = 1.88

$$\frac{\mathrm{e}^{-\frac{1}{x^5}}}{10} - \frac{\mathrm{e}^{\frac{1}{x^5}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(1/x^5)/x^6,x)
```

```
[Out] exp(-1/x^5)/10 - exp(1/x^5)/10
```

3.25 $\int \cosh\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=33

$$x\cosh\left(a+\frac{b}{x}\right)-b\mathrm{Chi}\left(\frac{b}{x}\right)\sinh(a)-b\cosh(a)\mathrm{Shi}\left(\frac{b}{x}\right)$$

```
[Out] x*cosh(a+b/x)-b*cosh(a)*Shi(b/x)-b*Chi(b/x)*sinh(a)
```

Rubi [A]
time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5411, 3378, 3384, 3379, 3382}

$$-b\sinh(a)\mathrm{Chi}\left(\frac{b}{x}\right)-b\cosh(a)\mathrm{Shi}\left(\frac{b}{x}\right)+x\cosh\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x],x]
```

```
[Out] x*Cosh[a + b/x] - b*CoshIntegral[b/x]*Sinh[a] - b*Cosh[a]*SinhIntegral[b/x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5411

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Cosh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \cosh\left(a+\frac{b}{x}\right)\,dx &= -\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= x\cosh\left(a+\frac{b}{x}\right) - b\text{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= x\cosh\left(a+\frac{b}{x}\right) - (b\cosh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, \frac{1}{x}\right) - (b\sinh(a))\text{Subst}\left(\int \right.\\
&= x\cosh\left(a+\frac{b}{x}\right) - b\text{Chi}\left(\frac{b}{x}\right)\sinh(a) - b\cosh(a)\text{Shi}\left(\frac{b}{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$
x\cosh\left(a+\frac{b}{x}\right) - b\text{Chi}\left(\frac{b}{x}\right)\sinh(a) - b\cosh(a)\text{Shi}\left(\frac{b}{x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x],x]
[Out] x*Cosh[a + b/x] - b*CoshIntegral[b/x]*Sinh[a] - b*Cosh[a]*SinhIntegral[b/x]
```

Maple [A]
time = 1.33, size = 56, normalized size = 1.70

method	result
risch	$-\frac{b\,\mathrm{e}^{-a}\,\mathrm{expIntegral}\left(1,\frac{b}{x}\right)}{2}+\frac{\mathrm{e}^{-\frac{ax+b}{x}}x}{2}+\frac{b\,\mathrm{e}^{a}\,\mathrm{expIntegral}\left(1,-\frac{b}{x}\right)}{2}+\frac{\mathrm{e}^{\frac{ax+b}{x}}x}{2}$
meijerg	$-\frac{i\sqrt{\pi}\,\cosh(a)b\left(\frac{4ix\cosh\left(\frac{b}{x}\right)}{b\sqrt{\pi}}-\frac{4i\,\mathrm{hyperbolicSineIntegral}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{4}-\frac{\sqrt{\pi}\,\sinh(a)b\left(\frac{4}{\sqrt{\pi}}-\frac{4x\sinh\left(\frac{b}{x}\right)}{\sqrt{\pi}\,b}+\frac{4\,\mathrm{hyperbolicCosineIntegral}\left(\frac{b}{x}\right.}{\sqrt{\pi}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x),x,method=_RETURNVERBOSE)
```

[Out] -1/2*b*exp(-a)*Ei(1,b/x)+1/2*exp(-(a*x+b)/x)*x+1/2*b*exp(a)*Ei(1,-b/x)+1/2*exp((a*x+b)/x)*x

Maxima [A]
time = 0.30, size = 37, normalized size = 1.12

$$\frac{1}{2}\left(\mathrm{Ei}\left(-\frac{b}{x}\right)e^{(-a)} - \mathrm{Ei}\left(\frac{b}{x}\right)e^{a}\right)b + x\cosh\left(a + \frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(a+b/x),x, algorithm="maxima")

[Out] 1/2*(Ei(-b/x)*e^(-a) - Ei(b/x)*e^a)*b + x*cosh(a + b/x)

Fricas [A]
time = 0.36, size = 58, normalized size = 1.76

$$-\frac{1}{2}\left(b\mathrm{Ei}\left(\frac{b}{x}\right) - b\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\cosh\left(a\right) + x\cosh\left(\frac{ax+b}{x}\right) - \frac{1}{2}\left(b\mathrm{Ei}\left(\frac{b}{x}\right) + b\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\sinh\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(a+b/x),x, algorithm="fricas")

[Out] -1/2*(b*Ei(b/x) - b*Ei(-b/x))*cosh(a) + x*cosh((a*x + b)/x) - 1/2*(b*Ei(b/x) + b*Ei(-b/x))*sinh(a)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(a + \frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(a+b/x),x)

[Out] Integral(cosh(a + b/x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(33) = 66.
time = 0.42, size = 173, normalized size = 5.24

$$\frac{ab^2\mathrm{Ei}\left(a - \frac{ax+b}{x}\right)e^{(-a)} - \frac{(ax+b)b^2\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}}{x} - b^2e^{\left(-\frac{ax+b}{x}\right)}}{2\left(a - \frac{ax+b}{x}\right)b} - \frac{ab^2\mathrm{Ei}\left(-a + \frac{ax+b}{x}\right)e^{a} - \frac{(ax+b)b^2\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}}{x} + b^2e^{\left(\frac{ax+b}{x}\right)}}{2\left(a - \frac{ax+b}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(a+b/x),x, algorithm="giac")

```
[Out] 1/2*(a*b^2*Ei(a - (a*x + b)/x)*e^(-a) - (a*x + b)*b^2*Ei(a - (a*x + b)/x)*e
^(-a)/x - b^2*e^(-(a*x + b)/x))/((a - (a*x + b)/x)*b) - 1/2*(a*b^2*Ei(-a +
(a*x + b)/x)*e^a - (a*x + b)*b^2*Ei(-a + (a*x + b)/x)*e^a/x + b^2*e^((a*x +
 b)/x))/((a - (a*x + b)/x)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cosh\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x),x)
```

```
[Out] int(cosh(a + b/x), x)
```

3.26 $\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x} dx$

Optimal. Leaf size=21

$$-\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[Out] -Chi(b/x)*cosh(a)-Shi(b/x)*sinh(a)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5427, 5425, 5424}

$$-\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x]/x,x]
```

```
[Out] -(Cosh[a]*CoshIntegral[b/x]) - Sinh[a]*SinhIntegral[b/x]
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x} dx &= \cosh(a)\int \frac{\cosh\left(\frac{b}{x}\right)}{x} dx+\sinh(a)\int \frac{\sinh\left(\frac{b}{x}\right)}{x} dx\\ &= -\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\sinh(a)\text{Shi}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$-\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x]/x,x]
[Out] -(Cosh[a]*CoshIntegral[b/x]) - Sinh[a]*SinhIntegral[b/x]
```

Maple [A]
time = 1.34, size = 27, normalized size = 1.29

method	result
risch	$\frac{e^{-a}\,\text{expIntegral}\left(1,\frac{b}{x}\right)}{2}+\frac{e^{a}\,\text{expIntegral}\left(1,-\frac{b}{x}\right)}{2}$
meijerg	$-\frac{\sqrt{\pi}\,\cosh(a)\left(\frac{2\,\text{hyperbolicCosineIntegral}\left(\frac{b}{x}\right)-2\ln\left(\frac{b}{x}\right)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma-2\ln(x)+2\ln(ib)}{\sqrt{\pi}}\right)}{2}-\text{hyperbolicSineIntegral}\left(\frac{b}{x}\right)\sinh($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(-a)*Ei(1,b/x)+1/2*exp(a)*Ei(1,-b/x)
```

Maxima [A]
time = 0.30, size = 24, normalized size = 1.14

$$-\frac{1}{2}\,\text{Ei}\left(-\frac{b}{x}\right)e^{(-a)}-\frac{1}{2}\,\text{Ei}\left(\frac{b}{x}\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x,x, algorithm="maxima")
[Out] -1/2*Ei(-b/x)*e^(-a) - 1/2*Ei(b/x)*e^a
```

Fricas [A]
time = 0.37, size = 39, normalized size = 1.86

$$-\frac{1}{2}\left(\text{Ei}\left(\frac{b}{x}\right)+\text{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a)-\frac{1}{2}\left(\text{Ei}\left(\frac{b}{x}\right)-\text{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(Ei(b/x) + Ei(-b/x))*cosh(a) - 1/2*(Ei(b/x) - Ei(-b/x))*sinh(a)
```

Sympy [A]
time = 0.64, size = 17, normalized size = 0.81

$$-\sinh\left(a\right)\operatorname{Shi}\left(\frac{b}{x}\right)-\cosh\left(a\right)\operatorname{Chi}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x,x)
```

```
[Out] -sinh(a)*Shi(b/x) - cosh(a)*Chi(b/x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.
time = 0.40, size = 43, normalized size = 2.05

$$-\frac{b\operatorname{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}+b\operatorname{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x,x, algorithm="giac")
```

```
[Out] -1/2*(b*Ei(a - (a*x + b)/x)*e^(-a) + b*Ei(-a + (a*x + b)/x)*e^a)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$-\cosh(a)\ \mathrm{coshint}\left(\frac{b}{x}\right)-\sinh(a)\ \mathrm{sinhint}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x)/x,x)
```

```
[Out] - cosh(a)*coshint(b/x) - sinh(a)*sinhint(b/x)
```

3.27 $\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=13

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] -sinh(a+b/x)/b
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5429, 2717}

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x]/x^2,x]
[Out] -(Sinh[a + b/x]/b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \cosh(a+bx)\,dx,x,\frac{1}{x}\right)\\ &= -\frac{\sinh\left(a+\frac{b}{x}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x]/x^2,x]
[Out] -(Sinh[a + b/x]/b)
```

Maple [A]
time = 0.41, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$	14
default	$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$	14
risch	$-\frac{e^{\frac{ax+b}{x}}}{2b}+\frac{e^{-\frac{ax+b}{x}}}{2b}$	33
meijerg	$-\frac{\cosh(a)\sinh\left(\frac{b}{x}\right)}{b}+\frac{\sqrt{\pi}\,\sinh(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{b}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x)/x^2,x,method=_RETURNVERBOSE)
[Out] -sinh(a+b/x)/b
```

Maxima [A]
time = 0.26, size = 13, normalized size = 1.00

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x^2,x, algorithm="maxima")
[Out] -sinh(a + b/x)/b
```

Fricas [A]
time = 0.57, size = 15, normalized size = 1.15

$$-\frac{\sinh\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x^2,x, algorithm="fricas")
[Out] -sinh((a*x + b)/x)/b
```

Sympy [A]
time = 0.33, size = 15, normalized size = 1.15

$$\begin{cases} -\frac{\sinh\left(a+\frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cosh(a)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x**2,x)
```

```
[Out] Piecewise((-sinh(a + b/x)/b, Ne(b, 0)), (-cosh(a)/x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.
time = 0.41, size = 29, normalized size = 2.23

$$-\frac{e^{\left(\frac{ax+b}{x}\right)} - e^{\left(-\frac{ax+b}{x}\right)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(e^((a*x + b)/x) - e^(-(a*x + b)/x))/b
```

Mupad [B]
time = 0.87, size = 13, normalized size = 1.00

$$-\frac{\sinh\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x)/x^2,x)
```

```
[Out] -sinh(a + b/x)/b
```

3.28 $\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=29

$$\frac{\cosh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx}$$

```
[Out] cosh(a+b/x)/b^2-sinh(a+b/x)/b/x
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5429, 3377, 2718}

$$\frac{\cosh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x]/x^3,x]
```

```
[Out] Cosh[a + b/x]/b^2 - Sinh[a + b/x]/(b*x)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sinh\left(a+\frac{b}{x}\right)}{bx} + \frac{\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x}\right)}{b}$$
$$= \frac{\cosh\left(a+\frac{b}{x}\right)}{b^2} - \frac{\sinh\left(a+\frac{b}{x}\right)}{bx}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{x\cosh\left(a+\frac{b}{x}\right) - b\sinh\left(a+\frac{b}{x}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x]/x^3,x]
[Out] (x*Cosh[a + b/x] - b*Sinh[a + b/x])/(b^2*x)
```

Maple [A]
time = 1.21, size = 44, normalized size = 1.52

method	result	size
derivativedivides	$-\frac{\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)-a\sinh\left(a+\frac{b}{x}\right)}{b^2}$	44
default	$-\frac{\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)-a\sinh\left(a+\frac{b}{x}\right)}{b^2}$	44
risch	$-\frac{(-x+b)\mathrm{e}^{\frac{ax+b}{x}}}{2b^2x}+\frac{(x+b)\mathrm{e}^{-\frac{ax+b}{x}}}{2b^2x}$	47
meijerg	$\frac{2\sqrt{\pi}\,\cosh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}}-\frac{b\sinh\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^2}-\frac{\sinh(a)\left(\frac{\cosh\left(\frac{b}{x}\right)b}{x}-\sinh\left(\frac{b}{x}\right)\right)}{b^2}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/b^2*((a+b/x)*sinh(a+b/x)-cosh(a+b/x)-a*sinh(a+b/x))
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.29, size = 47, normalized size = 1.62

$$\frac{1}{4}\,b\left(\frac{e^{(-a)}\Gamma\left(3,\frac{b}{x}\right)}{b^3}+\frac{e^a\Gamma\left(3,-\frac{b}{x}\right)}{b^3}\right)-\frac{\cosh\left(a+\frac{b}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*(e^(-a)*gamma(3, b/x)/b^3 + e^a*gamma(3, -b/x)/b^3) - 1/2*cosh(a + b/
x)/x^2
```

Fricas [A]
time = 0.44, size = 33, normalized size = 1.14

$$\frac{x\cosh\left(\frac{ax+b}{x}\right)-b\sinh\left(\frac{ax+b}{x}\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x^3,x, algorithm="fricas")
```

```
[Out] (x*cosh((a*x + b)/x) - b*sinh((a*x + b)/x))/(b^2*x)
```

Sympy [A]
time = 0.47, size = 29, normalized size = 1.00

$$\begin{cases} -\frac{\sinh\left(a+\frac{b}{x}\right)}{bx}+\frac{\cosh\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b\neq 0 \\ -\frac{\cosh(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x**3,x)
```

```
[Out] Piecewise((-sinh(a + b/x)/(b*x) + cosh(a + b/x)/b**2, Ne(b, 0)), (-cosh(a)/
(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(29) = 58$.
time = 0.41, size = 93, normalized size = 3.21

$$\frac{ae^{\left(\frac{ax+b}{x}\right)}-ae^{\left(-\frac{ax+b}{x}\right)}-\frac{(ax+b)e^{\left(\frac{ax+b}{x}\right)}}{x}+\frac{(ax+b)e^{\left(-\frac{ax+b}{x}\right)}}{x}+e^{\left(\frac{ax+b}{x}\right)}+e^{\left(-\frac{ax+b}{x}\right)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(a*e^((a*x + b)/x) - a*e^(-(a*x + b)/x) - (a*x + b)*e^((a*x + b)/x)/x +
 (a*x + b)*e^(-(a*x + b)/x)/x + e^((a*x + b)/x) + e^(-(a*x + b)/x))/b^2
```

Mupad [B]
time = 0.89, size = 29, normalized size = 1.00

$$\frac{\cosh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\sinh\left(a+\frac{b}{x}\right)}{b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x)/x^3,x)
```

```
[Out] cosh(a + b/x)/b^2 - sinh(a + b/x)/(b*x)
```

3.29 $\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4} \, dx$

Optimal. Leaf size=46

$$\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^3}-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[Out] 2*cosh(a+b/x)/b^2/x-2*sinh(a+b/x)/b^3-sinh(a+b/x)/b/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5429, 3377, 2717}

$$-\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^3}+\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x]/x^4,x]
```

```
[Out] (2*Cosh[a + b/x])/(b^2*x) - (2*Sinh[a + b/x])/b^3 - Sinh[a + b/x]/(b*x^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh\left(a+\frac{b}{x}\right)}{x^4}\,dx &= -\text{Subst}\left(\int x^2\cosh(a+bx)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\
&= \frac{2\cosh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}-\frac{2\text{Subst}\left(\int\cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}\\
&= \frac{2\cosh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^3}-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 39, normalized size = 0.85

$$
\frac{2bx\cosh\left(a+\frac{b}{x}\right)-\left(b^2+2x^2\right)\sinh\left(a+\frac{b}{x}\right)}{b^3x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x]/x^4,x]
```

```
[Out] (2*b*x*Cosh[a + b/x] - (b^2 + 2*x^2)*Sinh[a + b/x])/(b^3*x^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(46) = 92$.
time = 1.22, size = 94, normalized size = 2.04

method	result
risch	$-\frac{\left(b^2-2bx+2x^2\right)\mathrm{e}^{\frac{ax+b}{x}}}{2b^3x^2}+\frac{\left(b^2+2bx+2x^2\right)\mathrm{e}^{-\frac{ax+b}{x}}}{2b^3x^2}$
derivativedivides	$-\frac{a^2\sinh\left(a+\frac{b}{x}\right)-2a\left(\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)\right)+\left(a+\frac{b}{x}\right)^2\sinh\left(a+\frac{b}{x}\right)-2\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)+2\sinh(a+\frac{b}{x}}{b^3}$
default	$-\frac{a^2\sinh\left(a+\frac{b}{x}\right)-2a\left(\left(a+\frac{b}{x}\right)\sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)\right)+\left(a+\frac{b}{x}\right)^2\sinh\left(a+\frac{b}{x}\right)-2\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)+2\sinh(a+\frac{b}{x}}{b^3}$
meijerg	$-\frac{4i\sqrt{\pi}\cosh(a)\left(\frac{ib\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}-\frac{i\left(\frac{3b^2}{2x^2}+3\right)\sinh\left(\frac{b}{x}\right)}{6\sqrt{\pi}}\right)}{b^3}-\frac{4\sqrt{\pi}\sinh(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{b^2}{2x^2}+1\right)\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}}-\frac{b\sinh\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(a^2*sinh(a+b/x)-2*a*((a+b/x)*sinh(a+b/x)-cosh(a+b/x))+(a+b/x)^2*sin
h(a+b/x)-2*(a+b/x)*cosh(a+b/x)+2*sinh(a+b/x))
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 48, normalized size = 1.04

$$\frac{1}{6}\,b\left(\frac{e^{(-a)}\Gamma\left(4,\frac{b}{x}\right)}{b^4}-\frac{e^{a}\Gamma\left(4,-\frac{b}{x}\right)}{b^4}\right)-\frac{\cosh\left(a+\frac{b}{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*(e^(-a)*gamma(4, b/x)/b^4 - e^a*gamma(4, -b/x)/b^4) - 1/3*cosh(a + b/
x)/x^3
```

Fricas [A]
time = 0.39, size = 43, normalized size = 0.93

$$\frac{2\,bx\cosh\left(\frac{ax+b}{x}\right)-\left(b^2+2\,x^2\right)\sinh\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] (2*b*x*cosh((a*x + b)/x) - (b^2 + 2*x^2)*sinh((a*x + b)/x))/(b^3*x^2)
```

Sympy [A]
time = 0.69, size = 46, normalized size = 1.00

$$\begin{cases}-\frac{\sinh\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^3} & \text{for } b\neq 0\\ -\frac{\cosh\left(a\right)}{3x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x**4,x)
```

```
[Out] Piecewise((-sinh(a + b/x)/(b*x**2) + 2*cosh(a + b/x)/(b**2*x) - 2*sinh(a +
b/x)/b**3, Ne(b, 0)), (-cosh(a)/(3*x**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(46) = 92.
time = 0.41, size = 216, normalized size = 4.70

$$-\frac{a^2e^{\left(\frac{ax+b}{x}\right)}-a^2e^{\left(-\frac{ax+b}{x}\right)}+2\,ae^{\left(\frac{ax+b}{x}\right)}-\frac{2\,(ax+b)ae^{\left(\frac{ax+b}{x}\right)}}{x}+2\,ae^{\left(-\frac{ax+b}{x}\right)}+\frac{2\,(ax+b)ae^{\left(-\frac{ax+b}{x}\right)}}{x}+\frac{(ax+b)^2e^{\left(\frac{ax+b}{x}\right)}}{x^2}-\frac{2\,(ax+b)e^{\left(\frac{ax+b}{x}\right)}}{x}-\frac{(ax+b)^2e^{\left(-\frac{ax+b}{x}\right)}}{x^2}-\frac{2\,(ax+b)e^{\left(-\frac{ax+b}{x}\right)}}{x}+2\,e^{\left(\frac{ax+b}{x}\right)}-2\,e^{\left(-\frac{ax+b}{x}\right)}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*e^((a*x + b)/x) - a^2*e^(-(a*x + b)/x) + 2*a*e^((a*x + b)/x) - 2*
(a*x + b)*a*e^((a*x + b)/x)/x + 2*a*e^(-(a*x + b)/x) + 2*(a*x + b)*a*e^(-(a
*x + b)/x)/x + (a*x + b)^2*e^((a*x + b)/x)/x^2 - 2*(a*x + b)*e^((a*x + b)/x
)/x - (a*x + b)^2*e^(-(a*x + b)/x)/x^2 - 2*(a*x + b)*e^(-(a*x + b)/x)/x + 2
*e^((a*x + b)/x) - 2*e^(-(a*x + b)/x))/b^3
```

Mupad [B]
time = 0.93, size = 66, normalized size = 1.43

$$\frac{\mathrm{e}^{-a-\frac{b}{x}}\left(\frac{x}{b^2}+\frac{1}{2\,b}+\frac{x^2}{b^3}\right)}{x^2}-\frac{\mathrm{e}^{a+\frac{b}{x}}\left(\frac{1}{2\,b}-\frac{x}{b^2}+\frac{x^2}{b^3}\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x)/x^4,x)
```

```
[Out] (exp(- a - b/x)*(x/b^2 + 1/(2*b) + x^2/b^3))/x^2 - (exp(a + b/x)*(1/(2*b) -
 x/b^2 + x^2/b^3))/x^2
```

3.30 $\int \cosh\left(a+\frac{b}{x^2}\right)\,dx$

Optimal. Leaf size=67

$$x\cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{2}\sqrt{b}\,e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{b}\,e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}}{x}\right)$$

```
[Out] x*cosh(a+b/x^2)+1/2*erf(b^(1/2)/x)*b^(1/2)*Pi^(1/2)/exp(a)-1/2*exp(a)*erfi(
b^(1/2)/x)*b^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.625$, Rules used = {5411, 5435, 5406, 2235, 2236}

$$\frac{1}{2}\sqrt{\pi}\,e^{-a}\sqrt{b}\,\mathrm{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{\pi}\,e^{a}\sqrt{b}\,\mathrm{Erfi}\left(\frac{\sqrt{b}}{x}\right)+x\cosh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x^2],x]
```

```
[Out] x*Cosh[a + b/x^2] + (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]/x])/(2*E^a) - (Sqrt[b]*E^
a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/2
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5411

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Cosh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
```

```
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5435

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x
)^(m + 1)*(Cosh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cosh\left(a+\frac{b}{x^2}\right)\,dx &= -\text{Subst}\left(\int \frac{\cosh\left(a+bx^2\right)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= x\cosh\left(a+\frac{b}{x^2}\right) - (2b)\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= x\cosh\left(a+\frac{b}{x^2}\right) + b\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right) - b\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)\\
&= x\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{2}\sqrt{b}\,e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{1}{2}\sqrt{b}\,e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 71, normalized size = 1.06

$$
x\cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{2}\sqrt{b}\,\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)-\sinh(a))-\frac{1}{2}\sqrt{b}\,\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a))
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x^2],x]
```

```
[Out] x*Cosh[a + b/x^2] + (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]))/2
 - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]))/2
```

Maple [A]
time = 0.87, size = 70, normalized size = 1.04

method	result
risch	$\frac{\text{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{\pi}\,\mathrm{e}^{-a}\sqrt{b}}{2}+\frac{\mathrm{e}^{-a}\mathrm{e}^{-\frac{b}{x^2}}x}{2}+\frac{\mathrm{e}^{a}\mathrm{e}^{\frac{b}{x^2}}x}{2}-\frac{\mathrm{e}^{a}b\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{-b}}{x}\right)}{2\sqrt{-b}}$

meijerg	$-\frac{\sqrt{\pi}\cosh(a)\sqrt{2}\sqrt{ib}\left(-\frac{2x\sqrt{2}\,e^{\frac{b}{x^2}}}{\sqrt{\pi}\sqrt{ib}}-\frac{2x\sqrt{2}\,e^{-\frac{b}{x^2}}}{\sqrt{\pi}\sqrt{ib}}-\frac{2\sqrt{2}\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{\sqrt{ib}}+\frac{2\sqrt{2}\sqrt{b}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)}{\sqrt{ib}}\right)}{8}+ \frac{i\sqrt{\pi}\ \mathrm{si}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*erf(b^(1/2)/x)*Pi^(1/2)*exp(-a)*b^(1/2)+1/2*exp(-a)*exp(-b/x^2)*x+1/2*e
xp(a)*exp(b/x^2)*x-1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Maxima [A]
time = 0.29, size = 72, normalized size = 1.07

$$\frac{1}{2}\,b\left(\frac{\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{\frac{b}{x^2}}\right)-1\right)e^{(-a)}}{x\sqrt{\frac{b}{x^2}}}-\frac{\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{-\frac{b}{x^2}}\right)-1\right)e^{a}}{x\sqrt{-\frac{b}{x^2}}}\right)+x\cosh\left(a+\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(sqrt(pi)*(erf(sqrt(b/x^2)) - 1)*e^(-a)/(x*sqrt(b/x^2)) - sqrt(pi)*(e
rf(sqrt(-b/x^2)) - 1)*e^a/(x*sqrt(-b/x^2))) + x*cosh(a + b/x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(49) = 98$.
time = 0.40, size = 225, normalized size = 3.36

$$\frac{x\cosh\left(\frac{ax^2+b}{x^2}\right)^2+\sqrt{\pi}\left(\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right)+\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a)+(\cosh(a)+\sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\right)\sqrt{-b}\,\mathrm{erf}\left(\frac{\sqrt{-b}}{x}\right)+\sqrt{\pi}\left(\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right)-\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a)+(\cosh(a)-\sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\right)\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)+2\,x\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh\left(\frac{ax^2+b}{x^2}\right)+x\sinh\left(\frac{ax^2+b}{x^2}\right)^2+x}{2\left(\cosh\left(\frac{ax^2+b}{x^2}\right)+\sinh\left(\frac{ax^2+b}{x^2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) +
cosh((a*x^2 + b)/x^2)*sinh(a) + (cosh(a) + sinh(a))*sinh((a*x^2 + b)/x^2))*
sqrt(-b)*erf(sqrt(-b)/x) + sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) - cosh((
a*x^2 + b)/x^2)*sinh(a) + (cosh(a) - sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b
)*erf(sqrt(b)/x) + 2*x*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) + x*sinh
((a*x^2 + b)/x^2)^2 + x)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cosh\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x**2),x)
```

```
[Out] Integral(cosh(a + b/x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(cosh(a + b/x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(a+\frac{b}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x^2),x)
```

```
[Out] int(cosh(a + b/x^2), x)
```

3.31 $\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\cosh(a)\mathrm{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}\sinh(a)\mathrm{Shi}\left(\frac{b}{x^2}\right)$$

```
[Out] -1/2*Chi(b/x^2)*cosh(a)-1/2*Shi(b/x^2)*sinh(a)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5427, 5425, 5424}

$$-\frac{1}{2}\cosh(a)\mathrm{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}\sinh(a)\mathrm{Shi}\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x^2]/x,x]
```

```
[Out] -1/2*(Cosh[a]*CoshIntegral[b/x^2]) - (Sinh[a]*SinhIntegral[b/x^2])/2
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx &= \cosh(a)\int \frac{\cosh\left(\frac{b}{x^2}\right)}{x}\,dx+\sinh(a)\int \frac{\sinh\left(\frac{b}{x^2}\right)}{x}\,dx\\ &= -\frac{1}{2}\cosh(a)\mathrm{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}\sinh(a)\mathrm{Shi}\left(\frac{b}{x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}\left(-\cosh(a)\operatorname{Chi}\left(\frac{b}{x^2}\right)-\sinh(a)\operatorname{Shi}\left(\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x^2]/x,x]
[Out] (-(Cosh[a]*CoshIntegral[b/x^2]) - Sinh[a]*SinhIntegral[b/x^2])/2
```

Maple [A]
time = 0.78, size = 27, normalized size = 1.08

method	result	si
risch	$\frac{\mathrm{e}^{-a}\,\mathrm{expIntegral}\left(1,\frac{b}{x^2}\right)}{4}+\frac{\mathrm{e}^{a}\,\mathrm{expIntegral}\left(1,-\frac{b}{x^2}\right)}{4}$	2
meijerg	$-\frac{\sqrt{\pi}\,\cosh(a)\left(\frac{2\,\mathrm{hyperbolicCosineIntegral}\left(\frac{b}{x^2}\right)-2\ln\left(\frac{b}{x^2}\right)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma-4\ln(x)+2\ln(ib)}{\sqrt{\pi}}\right)}{4}-\frac{\mathrm{hyperbolicSineIntegral}\left(\frac{b}{x^2}\right)\sinh(a)}{2}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x^2)/x,x,method=_RETURNVERBOSE)
[Out] 1/4*exp(-a)*Ei(1,b/x^2)+1/4*exp(a)*Ei(1,-b/x^2)
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.96

$$-\frac{1}{4}\,\operatorname{Ei}\left(-\frac{b}{x^2}\right)e^{(-a)}-\frac{1}{4}\,\operatorname{Ei}\left(\frac{b}{x^2}\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x,x, algorithm="maxima")
[Out] -1/4*Ei(-b/x^2)*e^(-a) - 1/4*Ei(b/x^2)*e^a
```

Fricas [A]
time = 0.46, size = 39, normalized size = 1.56

$$-\frac{1}{4}\left(\operatorname{Ei}\left(\frac{b}{x^2}\right)+\operatorname{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh\left(a\right)-\frac{1}{4}\left(\operatorname{Ei}\left(\frac{b}{x^2}\right)-\operatorname{Ei}\left(-\frac{b}{x^2}\right)\right)\sinh\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(Ei(b/x^2) + Ei(-b/x^2))*cosh(a) - 1/4*(Ei(b/x^2) - Ei(-b/x^2))*sinh(a
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x**2)/x,x)
```

```
[Out] Integral(cosh(a + b/x**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(a + b/x^2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$-\frac{\cosh(a)\,\mathrm{coshint}\left(\frac{b}{x^2}\right)}{2}-\frac{\sinh(a)\,\mathrm{sinhint}\left(\frac{b}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x^2)/x,x)
```

```
[Out] - (cosh(a)*coshint(b/x^2))/2 - (sinh(a)*sinhint(b/x^2))/2
```

3.32 $\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

Optimal. Leaf size=57

$$-\frac{e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

```
[Out] -1/4*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)/b^(1/2)-1/4*exp(a)*erfi(b^(1/2)/x)*Pi^(
1/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5455, 5407, 2235, 2236}

$$-\frac{\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x^2]/x^2,x]
```

```
[Out] -1/4*(Sqrt[Pi]*Erf[Sqrt[b]/x])/(Sqrt[b]*E^a) - (E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x
])/(4*Sqrt[b])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5455

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> -Subst[Int[(a + b*Cosh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx &= -\mathrm{Subst}\left(\int \cosh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\ &= -\left(\frac{1}{2}\mathrm{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)\right)-\frac{1}{2}\mathrm{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.86

$$-\frac{\sqrt{\pi}\left(\mathrm{Erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)-\sinh(a))+\mathrm{Erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x^2]/x^2,x]
```

```
[Out] -1/4*(Sqrt[Pi]*(Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]) + Erfi[Sqrt[b]/x]*(Cosh[
a] + Sinh[a])))/Sqrt[b]
```

Maple [A]
time = 0.86, size = 44, normalized size = 0.77

method	result
risch	$-\frac{\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{\pi}\,\mathrm{e}^{-a}}{4\sqrt{b}}-\frac{\mathrm{e}^{a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-b}}{x}\right)}{4\sqrt{-b}}$
meijerg	$\frac{i\sqrt{\pi}\cosh(a)\sqrt{2}\sqrt{ib}\left(\frac{\sqrt{ib}\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{2\sqrt{b}}+\frac{\sqrt{ib}\sqrt{2}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)}{2\sqrt{b}}\right)}{4b}+\frac{\sqrt{\pi}\sinh(a)\sqrt{2}\sqrt{ib}\left(-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\mathrm{er}}{2b^{\frac{3}{2}}}\right.}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*erf(b^(1/2)/x)*Pi^(1/2)*exp(-a)/b^(1/2)-1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)
*erf((-b)^(1/2)/x)
```

Maxima [A]
time = 0.30, size = 63, normalized size = 1.11

$$\frac{1}{2}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{3}{2},\frac{b}{x^2}\right)}{x^3\left(\frac{b}{x^2}\right)^{\frac{3}{2}}}-\frac{e^{a}\Gamma\left(\frac{3}{2},-\frac{b}{x^2}\right)}{x^3\left(-\frac{b}{x^2}\right)^{\frac{3}{2}}}\right)-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(-a)*gamma(3/2, b/x^2)/(x^3*(b/x^2)^(3/2)) - e^a*gamma(3/2, -b/x^2
)/(x^3*(-b/x^2)^(3/2))) - cosh(a + b/x^2)/x
```

Fricas [A]
time = 0.42, size = 53, normalized size = 0.93

$$\frac{\sqrt{\pi}\,\sqrt{-b}\,(\cosh(a)+\sinh(a))\,\mathrm{erf}\left(\frac{\sqrt{-b}}{x}\right)-\sqrt{\pi}\,\sqrt{b}\,(\cosh(a)-\sinh(a))\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)/x) - sqrt(pi)*sqrt(
b)*(cosh(a) - sinh(a))*erf(sqrt(b)/x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x**2)/x**2,x)
```

```
[Out] Integral(cosh(a + b/x**2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(a + b/x^2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x^2)/x^2,x)
```

```
[Out] int(cosh(a + b/x^2)/x^2, x)
```

3.33 $\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3} dx$

Optimal. Leaf size=15

$$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[Out] -1/2*sinh(a+b/x^2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5429, 2717}

$$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x^2]/x^3,x]
```

```
[Out] -1/2*Sinh[a + b/x^2]/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^3} dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x^2]/x^3,x]
```

```
[Out] -1/2*Sinh[a + b/x^2]/b
```

Maple [A]
time = 0.41, size = 14, normalized size = 0.93

method	result	size
derivativedivides	$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$	14
default	$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b}$	14
risch	$-\frac{\mathrm{e}^{\frac{a\,x^2+b}{x^2}}}{4b}+\frac{\mathrm{e}^{-\frac{a\,x^2+b}{x^2}}}{4b}$	37
meijerg	$-\frac{\cosh(a)\sinh\left(\frac{b}{x^2}\right)}{2b}+\frac{\sqrt{\pi}\,\sinh(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{2b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*sinh(a+b/x^2)/b
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.87

$$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*sinh(a + b/x^2)/b
```

Fricas [A]
time = 0.45, size = 17, normalized size = 1.13

$$-\frac{\sinh\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*sinh((a*x^2 + b)/x^2)/b
```

Sympy [A]
time = 0.66, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b} & \text{for } b \neq 0 \\ -\frac{\cosh(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x**2)/x**3,x)
```

```
[Out] Piecewise((-sinh(a + b/x**2)/(2*b), Ne(b, 0)), (-cosh(a)/(2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.
time = 0.41, size = 33, normalized size = 2.20

$$-\frac{e^{\left(\frac{ax^2+b}{x^2}\right)} - e^{\left(-\frac{ax^2+b}{x^2}\right)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(e^((a*x^2 + b)/x^2) - e^(-(a*x^2 + b)/x^2))/b
```

Mupad [B]
time = 0.90, size = 13, normalized size = 0.87

$$-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x^2)/x^3,x)
```

```
[Out] -sinh(a + b/x^2)/(2*b)
```

3.34 $\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$

Optimal. Leaf size=75

$$-\frac{e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}$$

```
[Out] -1/2*sinh(a+b/x^2)/b/x-1/8*erf(b^(1/2)/x)*Pi^(1/2)/b^(3/2)/exp(a)+1/8*exp(a
)*erfi(b^(1/2)/x)*Pi^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5455, 5433, 5406, 2235, 2236}

$$-\frac{\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b/x^2]/x^4,x]
```

```
[Out] -1/8*(Sqrt[Pi]*Erf[Sqrt[b]/x])/(b^(3/2)*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x
])/(8*b^(3/2)) - Sinh[a + b/x^2]/(2*b*x)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5455

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> -Subst[Int[(a + b*Cosh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx &= -\text{Subst}\left(\int x^2\cosh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}+\frac{\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b}\\ &= -\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}-\frac{\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)}{4b}+\frac{\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)}{4b}\\ &= -\frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}+\frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2bx}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 74, normalized size = 0.99

$$\frac{\sqrt{\pi}\,x\text{Erf}\left(\frac{\sqrt{b}}{x}\right)(-\cosh(a)+\sinh(a))+\sqrt{\pi}\,x\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a)+\sinh(a))-4\sqrt{b}\,\sinh\left(a+\frac{b}{x^2}\right)}{8b^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b/x^2]/x^4,x]
```

```
[Out] (Sqrt[Pi]*x*Erf[Sqrt[b]/x]*(-Cosh[a] + Sinh[a]) + Sqrt[Pi]*x*Erfi[Sqrt[b]/x
]*(Cosh[a] + Sinh[a]) - 4*Sqrt[b]*Sinh[a + b/x^2])/(8*b^(3/2)*x)
```

Maple [A]
time = 0.88, size = 82, normalized size = 1.09

method	result

risch	$\frac{e^{-a}e^{-\frac{b}{x^2}}}{4bx} - \frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{\frac{3}{2}}} - \frac{e^{a}e^{\frac{b}{x^2}}}{4xb} + \frac{e^{a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-b}}{x}\right)}{8b\sqrt{-b}}$
meijerg	$\frac{\sqrt{\pi}\,\cosh(a)\sqrt{2}\,\sqrt{ib}\left(\frac{\sqrt{2}\,(ib)^{\frac{3}{2}}e^{\frac{b}{x^2}}}{4\sqrt{\pi}\,xb} - \frac{\sqrt{2}\,(ib)^{\frac{3}{2}}e^{-\frac{b}{x^2}}}{4\sqrt{\pi}\,xb} + \frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{\frac{3}{2}}} - \frac{(ib)^{\frac{3}{2}}\sqrt{2}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{\frac{3}{2}}}\right)}{2b^2} - i\sqrt{\pi}\,\sinh$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(-a)/b/x*exp(-b/x^2)-1/8*exp(-a)/b^(3/2)*Pi^(1/2)*erf(b^(1/2)/x)-1/4
*exp(a)*exp(b/x^2)/x/b+1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Maxima [A]
time = 0.31, size = 63, normalized size = 0.84

$$\frac{1}{6}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{5}{2},\frac{b}{x^2}\right)}{x^5\left(\frac{b}{x^2}\right)^{\frac{5}{2}}} - \frac{e^{a}\Gamma\left(\frac{5}{2},-\frac{b}{x^2}\right)}{x^5\left(-\frac{b}{x^2}\right)^{\frac{5}{2}}}\right) - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*(e^(-a)*gamma(5/2, b/x^2)/(x^5*(b/x^2)^(5/2)) - e^a*gamma(5/2, -b/x^2
)/(x^5*(-b/x^2)^(5/2))) - 1/3*cosh(a + b/x^2)/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(55) = 110$.
time = 0.51, size = 250, normalized size = 3.33

$$-\frac{2b\cosh\left(\frac{ax^2+b}{x^2}\right)^2 + \sqrt{\pi}\left(x\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right) + x\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a) + (x\cosh(a)+x\sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\right)\sqrt{-b}\,\mathrm{erf}\left(\frac{\sqrt{-b}}{x}\right) + \sqrt{\pi}\left(x\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right) - x\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a) + (x\cosh(a)-x\sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\right)\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right) + 4b\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh\left(\frac{ax^2+b}{x^2}\right) + 2b\sinh\left(\frac{ax^2+b}{x^2}\right)^2 - 2b}{8\left(b^2x\cosh\left(\frac{ax^2+b}{x^2}\right) + b^2x\sinh\left(\frac{ax^2+b}{x^2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/x^
2) + x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) + x*sinh(a))*sinh((a*x^2
+ b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) + sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/
x^2) - x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) - x*sinh(a))*sinh((a*x^
2 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) + 4*b*cosh((a*x^2 + b)/x^2)*sinh((a*x^2
 + b)/x^2) + 2*b*sinh((a*x^2 + b)/x^2)^2 - 2*b)/(b^2*x*cosh((a*x^2 + b)/x^2
) + b^2*x*sinh((a*x^2 + b)/x^2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x**2)/x**4,x)
```

```
[Out] Integral(cosh(a + b/x**2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b/x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(cosh(a + b/x^2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b/x^2)/x^4,x)
```

```
[Out] int(cosh(a + b/x^2)/x^4, x)
```

3.35 $\int \cosh\left(a+bx^n\right)\,dx$

Optimal. Leaf size=67

$$-\frac{e^a x\left(-bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{2n}-\frac{e^{-a}x\left(bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{2n}$$

```
[Out] -1/2*exp(a)*x*GAMMA(1/n,-b*x^n)/n/((-b*x^n)^(1/n))-1/2*x*GAMMA(1/n,b*x^n)/e
xp(a)/n/((b*x^n)^(1/n))
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{5415, 2239\}$

$$-\frac{e^a x\left(-bx^n\right)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n},-bx^n\right)}{2n}-\frac{e^{-a}x\left(bx^n\right)^{-1/n}\operatorname{Gamma}\left(\frac{1}{n},bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^n],x]
```

```
[Out] -1/2*(E^a*x*Gamma[n^(-1), -(b*x^n)])/(n*(-(b*x^n))^n^(-1)) - (x*Gamma[n^(-1
), b*x^n])/(2*E^a*n*(b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 5415

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh\left(a+bx^n\right)\,dx &= \frac{1}{2}\int e^{-a-bx^n}\,dx+\frac{1}{2}\int e^{a+bx^n}\,dx\\ &= -\frac{e^a x\left(-bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{2n}-\frac{e^{-a}x\left(bx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{2n}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 1.15

$$-\frac{x\left(-b^2x^{2n}\right)^{-1/n}\left(\left(-bx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},bx^n\right)\left(\cosh(a)-\sinh(a)\right)+\left(bx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-bx^n\right)\left(\cosh(a)+\sinh(a)\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^n],x]
```

```
[Out] -1/2*(x*((-(b*x^n))^n^(-1)*Gamma[n^(-1), b*x^n]*(Cosh[a] - Sinh[a]) + (b*x^
n)^n^(-1)*Gamma[n^(-1), -(b*x^n)]*(Cosh[a] + Sinh[a])))/(n*(-(b^2*x^(2*n)))
^n^(-1))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.45, size = 74, normalized size = 1.10

method	result	siz
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{2n}\right], \left[\frac{1}{2}, 1+\frac{1}{2n}\right], \frac{x^{2n}b^2}{4}\right)\cosh(a) + \frac{x^{1+n}b\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{1}{2n}\right], \left[\frac{3}{2}, \frac{3}{2}+\frac{1}{2n}\right], \frac{x^{2n}b^2}{4}\right)\sinh(a)}{1+n}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/2/n],[1/2,1+1/2/n],1/4*x^(2*n)*b^2)*cosh(a)+1/(1+n)*x^(1+n)*
b*hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],1/4*x^(2*n)*b^2)*sinh(a)
```

Maxima [A]
time = 0.07, size = 61, normalized size = 0.91

$$-\frac{xe^{(-a)}\Gamma\left(\frac{1}{n}, bx^n\right)}{2\,(bx^n)^{\left(\frac{1}{n}\right)}\,n} - \frac{xe^{a}\Gamma\left(\frac{1}{n}, -bx^n\right)}{2\,(-bx^n)^{\left(\frac{1}{n}\right)}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/2*x*e^(-a)*gamma(1/n, b*x^n)/((b*x^n)^(1/n)*n) - 1/2*x*e^a*gamma(1/n, -b
*x^n)/((-b*x^n)^(1/n)*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a + bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x**n),x)
```

```
[Out] Integral(cosh(a + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a + b\,x^n)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^n),x)
```

```
[Out] int(cosh(a + b*x^n), x)
```

3.36 $\int \frac{\cosh(a+bx^n)}{x} \, dx$

Optimal. Leaf size=25

$$\frac{\cosh(a)\text{Chi}(bx^n)}{n}+\frac{\sinh(a)\text{Shi}(bx^n)}{n}$$

```
[Out] Chi(b*x^n)*cosh(a)/n+Shi(b*x^n)*sinh(a)/n
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5427, 5425, 5424}

$$\frac{\cosh(a)\text{Chi}(bx^n)}{n}+\frac{\sinh(a)\text{Shi}(bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^n]/x,x]
```

```
[Out] (Cosh[a]*CoshIntegral[b*x^n])/n + (Sinh[a]*SinhIntegral[b*x^n])/n
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(a+bx^n\right)}{x} \, dx &= \cosh(a)\int \frac{\cosh\left(bx^n\right)}{x} \, dx+\sinh(a)\int \frac{\sinh\left(bx^n\right)}{x} \, dx\\ &= \frac{\cosh(a)\text{Chi}(bx^n)}{n}+\frac{\sinh(a)\text{Shi}(bx^n)}{n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.92

$$\frac{\cosh(a)\mathrm{Chi}(bx^n) + \sinh(a)\mathrm{Shi}(bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^n]/x,x]
[Out] (Cosh[a]*CoshIntegral[b*x^n] + Sinh[a]*SinhIntegral[b*x^n])/n
```

Maple [A]
time = 1.25, size = 33, normalized size = 1.32

method	result	size
risch	$-\frac{\mathrm{e}^{-a} \operatorname{expIntegral}(1,b\,x^{n})}{2n} - \frac{\mathrm{e}^{a} \operatorname{expIntegral}(1,-b\,x^{n})}{2n}$	33
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\,\mathrm{hyperbolicCosineIntegral}(b\,x^{n})-2\ln(b\,x^{n})-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2n\ln(x)+2\ln(ib)}{\sqrt{\pi}}\right)\cosh(a)}{2n} + \frac{\mathrm{hyperbolicSineIntegral}(b\,x^{n})\sinh(a)}{n}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*x^n)/x,x,method=_RETURNVERBOSE)
[Out] -1/2/n*exp(-a)*Ei(1,b*x^n)-1/2/n*exp(a)*Ei(1,-b*x^n)
```

Maxima [A]
time = 0.33, size = 30, normalized size = 1.20

$$\frac{\mathrm{Ei}(-bx^n)\, e^{(-a)}}{2\,n} + \frac{\mathrm{Ei}(bx^n)\, e^{a}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)/x,x, algorithm="maxima")
[Out] 1/2*Ei(-b*x^n)*e^(-a)/n + 1/2*Ei(b*x^n)*e^a/n
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.
time = 0.45, size = 54, normalized size = 2.16

$$\frac{(\cosh{(a)} + \sinh{(a)})\mathrm{Ei}(b\cosh{(n\log{(x)})} + b\sinh{(n\log{(x)})}) + (\cosh{(a)} - \sinh{(a)})\mathrm{Ei}(-b\cosh{(n\log{(x)})} - b\sinh{(n\log{(x)})})}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + (cosh(a)
 - sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))))/n
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x**n)/x,x)
```

```
[Out] Integral(cosh(a + b*x**n)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x^n + a)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cosh(a+b\,x^{n})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^n)/x,x)
```

```
[Out] int(cosh(a + b*x^n)/x, x)
```

3.37 $\int \cosh^2(a+bx^n)\,dx$

Optimal. Leaf size=89

$$\frac{x}{2}-\frac{2^{-2-\frac{1}{n}}e^{2a}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{2^{-2-\frac{1}{n}}e^{-2a}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2bx^n\right)}{n}$$

```
[Out] 1/2*x-2^(-2-1/n)*exp(2*a)*x*GAMMA(1/n,-2*b*x^n)/n/((-b*x^n)^(1/n))-2^(-2-1/
n)*x*GAMMA(1/n,2*b*x^n)/exp(2*a)/n/((b*x^n)^(1/n))
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5417, 5415, 2239}

$$-\frac{e^{2a}2^{-\frac{1}{n}-2}x(-bx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{1}{n}-2}x(bx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},2bx^n\right)}{n}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^n]^2,x]
```

```
[Out] x/2 - (2^(-2 - n^(-1))*E^(2*a)*x*Gamma[n^(-1), -2*b*x^n])/(n*(-(b*x^n))^n^(
-1)) - (2^(-2 - n^(-1))*x*Gamma[n^(-1), 2*b*x^n])/(E^(2*a)*n*(b*x^n)^n^(-1)
)
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 5415

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 5417

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cosh^2(a+bx^n)\,dx &= \int \left(\frac{1}{2}+\frac{1}{2}\cosh(2a+2bx^n)\right)dx \\
&= \frac{x}{2}+\frac{1}{2}\int \cosh(2a+2bx^n)\,dx \\
&= \frac{x}{2}+\frac{1}{4}\int e^{-2a-2bx^n}\,dx+\frac{1}{4}\int e^{2a+2bx^n}\,dx \\
&= \frac{x}{2}-\frac{2^{-2-\frac{1}{n}}e^{2a}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{2^{-2-\frac{1}{n}}e^{-2a}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2bx^n\right)}{n}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 81, normalized size = 0.91

$$
-\frac{x\left(-2n+2^{-1/n}e^{2a}(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2bx^n\right)+2^{-1/n}e^{-2a}(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2bx^n\right)\right)}{4n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^n]^2,x]
```

```
[Out] -1/4*(x*(-2*n + (E^(2*a)*Gamma[n^(-1), -2*b*x^n])/(2^n^(-1)*(-(b*x^n))^n^(-
1)) + Gamma[n^(-1), 2*b*x^n]/(2^n^(-1)*E^(2*a)*(b*x^n)^n^(-1))))/n
```

Maple [F]
time = 1.05, size = 0, normalized size = 0.00

$$
\int \cosh^2(a+b\,x^n)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*x^n)^2,x)
```

```
[Out] int(cosh(a+b*x^n)^2,x)
```

Maxima [A]
time = 0.08, size = 68, normalized size = 0.76

$$
\frac{1}{2}\,x-\frac{xe^{(-2\,a)}\Gamma\left(\frac{1}{n},2\,bx^n\right)}{4\,(2\,bx^n)^{\left(\frac{1}{n}\right)}\,n}-\frac{xe^{(2\,a)}\Gamma\left(\frac{1}{n},-2\,bx^n\right)}{4\,(-2\,bx^n)^{\left(\frac{1}{n}\right)}\,n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*x*e^(-2*a)*gamma(1/n, 2*b*x^n)/((2*b*x^n)^(1/n)*n) - 1/4*x*e^(2
*a)*gamma(1/n, -2*b*x^n)/((-2*b*x^n)^(1/n)*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x^n + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^2(a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x**n)**2,x)
```

```
[Out] Integral(cosh(a + b*x**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a + b\,x^n)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^n)^2,x)
```

```
[Out] int(cosh(a + b*x^n)^2, x)
```

3.38 $\int \frac{\cosh^2(a+bx^n)}{x} \, dx$

Optimal. Leaf size=43

$$\frac{\cosh(2a)\text{Chi}(2bx^n)}{2n} + \frac{\log(x)}{2} + \frac{\sinh(2a)\text{Shi}(2bx^n)}{2n}$$

```
[Out] 1/2*Chi(2*b*x^n)*cosh(2*a)/n+1/2*ln(x)+1/2*Shi(2*b*x^n)*sinh(2*a)/n
```

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5471, 5427, 5425, 5424}

$$\frac{\cosh(2a)\text{Chi}(2bx^n)}{2n} + \frac{\sinh(2a)\text{Shi}(2bx^n)}{2n} + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^n]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^n])/(2*n) + Log[x]/2 + (Sinh[2*a]*SinhIntegra
l[2*b*x^n])/(2*n)
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5471

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx^n)}{x}\,dx &= \int \left(\frac{1}{2x}+\frac{\cosh(2a+2bx^n)}{2x}\right)dx\\ &= \frac{\log(x)}{2}+\frac{1}{2}\int \frac{\cosh(2a+2bx^n)}{x}\,dx\\ &= \frac{\log(x)}{2}+\frac{1}{2}\cosh(2a)\int \frac{\cosh(2bx^n)}{x}\,dx+\frac{1}{2}\sinh(2a)\int \frac{\sinh(2bx^n)}{x}\,dx\\ &= \frac{\cosh(2a)\mathrm{Chi}(2bx^n)}{2n}+\frac{\log(x)}{2}+\frac{\sinh(2a)\mathrm{Shi}(2bx^n)}{2n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.84

$$\frac{\cosh(2a)\mathrm{Chi}(2bx^n)+n\log(x)+\sinh(2a)\mathrm{Shi}(2bx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^n]^2/x,x]
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^n] + n*Log[x] + Sinh[2*a]*SinhIntegral[2*b*x^
n])/(2*n)
```

Maple [A]
time = 5.18, size = 40, normalized size = 0.93

method	result	size
risch	$\frac{\ln(x)}{2}-\frac{\mathrm{e}^{-2a}\,\mathrm{expIntegral}(1,2b\,x^n)}{4n}-\frac{\mathrm{e}^{2a}\,\mathrm{expIntegral}(1,-2b\,x^n)}{4n}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*x^n)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)-1/4/n*exp(-2*a)*Ei(1,2*b*x^n)-1/4/n*exp(2*a)*Ei(1,-2*b*x^n)
```

Maxima [A]
time = 0.33, size = 37, normalized size = 0.86

$$\frac{\mathrm{Ei}(2\,bx^n)\,e^{(2\,a)}}{4\,n}+\frac{\mathrm{Ei}(-2\,bx^n)\,e^{(-2\,a)}}{4\,n}+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^2/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*b*x^n)*e^(2*a)/n + 1/4*Ei(-2*b*x^n)*e^(-2*a)/n + 1/2*log(x)
```

Fricas [A]
time = 0.60, size = 69, normalized size = 1.60

$$\frac{(\cosh(2a)+\sinh(2a))\mathrm{Ei}(2b\cosh(n\log(x))+2b\sinh(n\log(x)))+(\cosh(2a)-\sinh(2a))\mathrm{Ei}(-2b\cosh(n\log(x))-2b\sinh(n\log(x)))+2n\log(x)}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*((cosh(2*a) + sinh(2*a))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) +
(cosh(2*a) - sinh(2*a))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) + 2*n*
log(x))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^2(a+bx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x**n)**2/x,x)
```

```
[Out] Integral(cosh(a + b*x**n)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x^n + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\cosh(a+b\,x^n)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^n)^2/x,x)
```

```
[Out] int(cosh(a + b*x^n)^2/x, x)
```

3.39 $\int \cosh^3(a + bx^n)\, dx$

Optimal. Leaf size=150

$$-\frac{3^{-1/n}e^{3a}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}-\frac{3e^{a}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},bx^n\right)}{8n}-\frac{3^{-1/n}e^{-3a}x(}{}$$

```
[Out] -1/8*exp(3*a)*x*GAMMA(1/n,-3*b*x^n)/(3^(1/n))/n/((-b*x^n)^(1/n))-3/8*exp(a)
*x*GAMMA(1/n,-b*x^n)/n/((-b*x^n)^(1/n))-3/8*x*GAMMA(1/n,b*x^n)/exp(a)/n/((b
*x^n)^(1/n))-1/8*x*GAMMA(1/n,3*b*x^n)/(3^(1/n))/exp(3*a)/n/((b*x^n)^(1/n))
```

Rubi [A]
time = 0.06, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5417, 5415, 2239}

$$-\frac{e^{3a}3^{-1/n}x(-bx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-3bx^n\right)}{8n}-\frac{3e^{a}x(-bx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x(bx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},bx^n\right)}{8n}-\frac{e^{-3a}3^{-1/n}x(bx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},3bx^n\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(E^(3*a)*x*Gamma[n^(-1), -3*b*x^n])/(3^n^(-1)*n*(-(b*x^n))^n^(-1)) - (
3*E^a*x*Gamma[n^(-1), -(b*x^n)])/(8*n*(-(b*x^n))^n^(-1)) - (3*x*Gamma[n^(-1
), b*x^n])/(8*E^a*n*(b*x^n)^n^(-1)) - (x*Gamma[n^(-1), 3*b*x^n])/(8*3^n^(-1
)*E^(3*a)*n*(b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 5415

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 5417

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

Rule steps

$$
\begin{aligned}
\int \cosh^3\left(a+bx^n\right)\,dx &= \int \left(\frac{3}{4}\cosh\left(a+bx^n\right)+\frac{1}{4}\cosh\left(3a+3bx^n\right)\right)\,dx \\
&= \frac{1}{4}\int \cosh\left(3a+3bx^n\right)\,dx+\frac{3}{4}\int \cosh\left(a+bx^n\right)\,dx \\
&= \frac{1}{8}\int e^{-3a-3bx^n}\,dx+\frac{1}{8}\int e^{3a+3bx^n}\,dx+\frac{3}{8}\int e^{-a-bx^n}\,dx+\frac{3}{8}\int e^{a+bx^n}\,dx \\
&= -\frac{3^{-1/n}e^{3a}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}-\frac{3e^{a}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x(bx^n)}{}
\end{aligned}
$$

Mathematica [A]
time = 1.34, size = 138, normalized size = 0.92

$$
-\frac{3^{-1/n}e^{-3a}x(-b^2x^{2n})^{-1/n}\left(e^{6a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-3bx^n\right)+3^{1+\frac{1}{n}}e^{4a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-bx^n\right)+(-bx^n)^{\frac{1}{n}}\left(3^{1+\frac{1}{n}}e^{2a}\Gamma\left(\frac{1}{n},bx^n\right)+\Gamma\left(\frac{1}{n},3bx^n\right)\right)\right)}{8n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(x*(E^(6*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -3*b*x^n] + 3^(1 + n^(-1))*E^
(4*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -(b*x^n)] + (-(b*x^n))^n^(-1)*(3^(1 + n^
(-1))*E^(2*a)*Gamma[n^(-1), b*x^n] + Gamma[n^(-1), 3*b*x^n])))/(3^n^(-1)*E^
(3*a)*n*(-(b^2*x^(2*n)))^n^(-1))
```

Maple [F]
time = 1.01, size = 0, normalized size = 0.00

$$
\int \cosh^3\left(a+b\,x^n\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*x^n)^3,x)
```

```
[Out] int(cosh(a+b*x^n)^3,x)
```

Maxima [A]
time = 0.09, size = 125, normalized size = 0.83

$$
-\frac{xe^{(-3\,a)}\Gamma\left(\frac{1}{n},3\,bx^n\right)}{8\,\left(3\,bx^n\right)^{\left(\frac{1}{n}\right)}\,n}-\frac{3\,xe^{(-a)}\Gamma\left(\frac{1}{n},bx^n\right)}{8\,\left(bx^n\right)^{\left(\frac{1}{n}\right)}\,n}-\frac{3\,xe^{a}\Gamma\left(\frac{1}{n},-bx^n\right)}{8\,\left(-bx^n\right)^{\left(\frac{1}{n}\right)}\,n}-\frac{xe^{(3\,a)}\Gamma\left(\frac{1}{n},-3\,bx^n\right)}{8\,\left(-3\,bx^n\right)^{\left(\frac{1}{n}\right)}\,n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -1/8*x*e^(-3*a)*gamma(1/n, 3*b*x^n)/((3*b*x^n)^(1/n)*n) - 3/8*x*e^(-a)*gamm
a(1/n, b*x^n)/((b*x^n)^(1/n)*n) - 3/8*x*e^a*gamma(1/n, -b*x^n)/((-b*x^n)^(1
/n)*n) - 1/8*x*e^(3*a)*gamma(1/n, -3*b*x^n)/((-3*b*x^n)^(1/n)*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^3,x, algorithm="fricas")
[Out] integral(cosh(b*x^n + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^3\left(a + bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x**n)**3,x)
[Out] Integral(cosh(a + b*x**n)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(cosh(b*x^n + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{cosh}\left(a+b\,x^n\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^n)^3,x)
[Out] int(cosh(a + b*x^n)^3, x)
```

3.40 $\int \frac{\cosh^3(a+bx^n)}{x} \, dx$

Optimal. Leaf size=67

$$\frac{3\cosh(a)\text{Chi}(bx^n)}{4n}+\frac{\cosh(3a)\text{Chi}(3bx^n)}{4n}+\frac{3\sinh(a)\text{Shi}(bx^n)}{4n}+\frac{\sinh(3a)\text{Shi}(3bx^n)}{4n}$$

```
[Out] 3/4*Chi(b*x^n)*cosh(a)/n+1/4*Chi(3*b*x^n)*cosh(3*a)/n+3/4*Shi(b*x^n)*sinh(a
)/n+1/4*Shi(3*b*x^n)*sinh(3*a)/n
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5471, 5427, 5425, 5424}

$$\frac{3\cosh(a)\text{Chi}(bx^n)}{4n}+\frac{\cosh(3a)\text{Chi}(3bx^n)}{4n}+\frac{3\sinh(a)\text{Shi}(bx^n)}{4n}+\frac{\sinh(3a)\text{Shi}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x^n]^3/x,x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x^n])/(4*n) + (Cosh[3*a]*CoshIntegral[3*b*x^n])/(
4*n) + (3*Sinh[a]*SinhIntegral[b*x^n])/(4*n) + (Sinh[3*a]*SinhIntegral[3*b*
x^n])/(4*n)
```

Rule 5424

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5471

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx^n)}{x}\,dx &= \int \left(\frac{3\cosh(a+bx^n)}{4x}+\frac{\cosh(3a+3bx^n)}{4x}\right)dx\\
&= \frac{1}{4}\int \frac{\cosh(3a+3bx^n)}{x}\,dx+\frac{3}{4}\int \frac{\cosh(a+bx^n)}{x}\,dx\\
&= \frac{1}{4}(3\cosh(a))\int \frac{\cosh(bx^n)}{x}\,dx+\frac{1}{4}\cosh(3a)\int \frac{\cosh(3bx^n)}{x}\,dx+\frac{1}{4}(3\sinh(a))\int \frac{\sinh
\\
&= \frac{3\cosh(a)\mathrm{Chi}(bx^n)}{4n}+\frac{\cosh(3a)\mathrm{Chi}(3bx^n)}{4n}+\frac{3\sinh(a)\mathrm{Shi}(bx^n)}{4n}+\frac{\sinh(3a)\mathrm{Shi}(3bx^n)}{4n}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 0.78

$$\frac{3\cosh(a)\mathrm{Chi}(bx^n)+\cosh(3a)\mathrm{Chi}(3bx^n)+3\sinh(a)\mathrm{Shi}(bx^n)+\sinh(3a)\mathrm{Shi}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x^n]^3/x,x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x^n] + Cosh[3*a]*CoshIntegral[3*b*x^n] + 3*Sinh[a
]*SinhIntegral[b*x^n] + Sinh[3*a]*SinhIntegral[3*b*x^n])/(4*n)
```

Maple [A]
time = 4.84, size = 67, normalized size = 1.00

method	result	size
risch	$-\frac{e^{-3a}\,\mathrm{expIntegral}(1,3b\,x^n)}{8n}-\frac{3\,e^{-a}\,\mathrm{expIntegral}(1,b\,x^n)}{8n}-\frac{3\,e^{a}\,\mathrm{expIntegral}(1,-b\,x^n)}{8n}-\frac{e^{3a}\,\mathrm{expIntegral}(1,-3b\,x^n)}{8n}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*x^n)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/n*exp(-3*a)*Ei(1,3*b*x^n)-3/8/n*exp(-a)*Ei(1,b*x^n)-3/8/n*exp(a)*Ei(1,
-b*x^n)-1/8/n*exp(3*a)*Ei(1,-3*b*x^n)
```

Maxima [A]
time = 0.34, size = 62, normalized size = 0.93

$$\frac{\mathrm{Ei}(3\,bx^n)\,e^{(3\,a)}}{8\,n}+\frac{3\,\mathrm{Ei}(-bx^n)\,e^{(-a)}}{8\,n}+\frac{\mathrm{Ei}(-3\,bx^n)\,e^{(-3\,a)}}{8\,n}+\frac{3\,\mathrm{Ei}(bx^n)\,e^{a}}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x^n)*e^(3*a)/n + 3/8*Ei(-b*x^n)*e^(-a)/n + 1/8*Ei(-3*b*x^n)*e^(-
3*a)/n + 3/8*Ei(b*x^n)*e^a/n
```

Fricas [A]
time = 0.39, size = 114, normalized size = 1.70

$$\frac{(\cosh(3a)+\sinh(3a))\mathrm{Ei}(3b\cosh(n\log(x))+3b\sinh(n\log(x)))+3(\cosh(a)+\sinh(a))\mathrm{Ei}(b\cosh(n\log(x))+b\sinh(n\log(x)))+3(\cosh(a)-\sinh(a))\mathrm{Ei}(-b\cosh(n\log(x))-b\sinh(n\log(x)))+(\cosh(3a)-\sinh(3a))\mathrm{Ei}(-3b\cosh(n\log(x))-3b\sinh(n\log(x)))}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] 1/8*((cosh(3*a) + sinh(3*a))*Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) +
3*(cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + 3*(cosh(a)
- sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))) + (cosh(3*a) - sinh(3*a
))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*log(x))))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x**n)**3/x,x)
```

```
[Out] Integral(cosh(a + b*x**n)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x^n + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh(a+b\,x^n)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^n)^3/x,x)
```

```
[Out] int(cosh(a + b*x^n)^3/x, x)
```

3.41 $\int (ex)^m (b\cosh(c+dx^n))^p \, dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left((ex)^m (b\cosh(c+dx^n))^p, x\right)$$

```
[Out] Unintegrable((e*x)^m*(b*cosh(c+d*x^n))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (b\cosh(c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m*(b*Cosh[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(b*Cosh[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (b\cosh(c+dx^n))^p \, dx = \int (ex)^m (b\cosh(c+dx^n))^p \, dx$$

Mathematica [A]
time = 3.74, size = 0, normalized size = 0.00

$$\int (ex)^m (b\cosh(c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m*(b*Cosh[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(b*Cosh[c + d*x^n])^p, x]
```

Maple [A]
time = 0.57, size = 0, normalized size = 0.00

$$\int (ex)^m (b\cosh(c+d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*cosh(c+d*x^n))^p,x)
```

[Out] int((e*x)^m*(b*cosh(c+d*x^n))^p,x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(b*cosh(c+d*x^n))^p,x, algorithm="maxima")

[Out] integrate((b*cosh(d*x^n + c))^p*(x*e)^m, x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(b*cosh(c+d*x^n))^p,x, algorithm="fricas")

[Out] integral((b*cosh(d*x^n + c))^p*(x*e)^m, x)

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cosh\left(c+dx^{n}\right)\right)^{p}\left(ex\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m*(b*cosh(c+d*x**n))**p,x)

[Out] Integral((b*cosh(c + d*x**n))**p*(e*x)**m, x)

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^m*(b*cosh(c+d*x^n))^p,x, algorithm="giac")

[Out] integrate((e*x)^m*(b*cosh(d*x^n + c))^p, x)

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int {\left(e\,x\right)}^m\,{\left(b\,\mathrm{cosh}\left(c+d\,x^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^m*(b*cosh(c + d*x^n))^p,x)

[Out] int((e*x)^m*(b*cosh(c + d*x^n))^p, x)

3.42 $\int (ex)^m (a + b\cosh(c + dx^n))^p \, dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left((ex)^m (a + b\cosh(c + dx^n))^p, x\right)$$

```
[Out] Unintegrable((e*x)^m*(a+b*cosh(c+d*x^n))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a + b\cosh(c + dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Cosh[c + d*x^n])^p,x]
[Out] Defer[Int][(e*x)^m*(a + b*Cosh[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (a + b\cosh(c + dx^n))^p \, dx = \int (ex)^m (a + b\cosh(c + dx^n))^p \, dx$$

Mathematica [A]
time = 5.68, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\cosh(c + dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Cosh[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Cosh[c + d*x^n])^p, x]
```

Maple [A]
time = 0.78, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\cosh(c + d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*cosh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^m*(a+b*cosh(c+d*x^n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x^n + c) + a)^p*(x*e)^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x^n + c) + a)^p*(x*e)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\cosh(c + dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*cosh(c+d*x**n))**p,x)
```

```
[Out] Integral((e*x)**m*(a + b*cosh(c + d*x**n))**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*(b*cosh(d*x^n + c) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e\,x)^m\,(a + b\cosh(c + d\,x^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*cosh(c + d*x^n))^p,x)
[Out] int((e*x)^m*(a + b*cosh(c + d*x^n))^p, x)
```

3.43 $\int (ex)^{-1+n} (b\cosh(c+dx^n))^p \, dx$

Optimal. Leaf size=95

$$-\frac{x^{-n}(ex)^n (b\cosh(c+dx^n))^{1+p} \, {}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\cosh^2(c+dx^n)\right)\sinh(c+dx^n)}{bden(1+p)\sqrt{-\sinh^2(c+dx^n)}}$$

```
[Out] -(e*x)^n*(b*cosh(c+d*x^n))^(1+p)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],cos
h(c+d*x^n)^2)*sinh(c+d*x^n)/b/d/e/n/(1+p)/(x^n)/(-sinh(c+d*x^n)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5431, 5429, 2722}

$$-\frac{x^{-n}(ex)^n \sinh(c+dx^n) (b\cosh(c+dx^n))^{p+1} \, {}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cosh^2(dx^n+c)\right)}{bden(p+1)\sqrt{-\sinh^2(c+dx^n)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(b*Cosh[c + d*x^n])^p,x]
```

```
[Out] -(((e*x)^n*(b*Cosh[c + d*x^n])^(1 + p)*Hypergeometric2F1[1/2, (1 + p)/2, (3
 + p)/2, Cosh[c + d*x^n]^2]*Sinh[c + d*x^n])/(b*d*e*n*(1 + p)*x^n*Sqrt[-Sin
h[c + d*x^n]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5431

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
```

```
tegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int (ex)^{-1+n} (b\cosh(c+dx^n))^p \, dx &= \frac{(x^{-n}(ex)^n) \int x^{-1+n}(b\cosh(c+dx^n))^p \, dx}{e} \\
&= \frac{(x^{-n}(ex)^n)\,\mathrm{Subst}\left(\int (b\cosh(c+dx))^p \, dx, x, x^n\right)}{en} \\
&= -\frac{x^{-n}(ex)^n (b\cosh(c+dx^n))^{1+p} \, {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \cosh^2(c+dx^n)\right) \sinh(c
\end{aligned}
$$

$$
bden(1+p)\sqrt{-\sinh^2(c+dx^n)}
$$

Mathematica [A]
time = 0.11, size = 94, normalized size = 0.99

$$
-\frac{x^{-n}(ex)^n (b\cosh(c+dx^n))^p \, {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \cosh^2(c+dx^n)\right) \sinh(2(c+dx^n))}{2den(1+p)\sqrt{-\sinh^2(c+dx^n)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(-1 + n)*(b*Cosh[c + d*x^n])^p,x]
```

```
[Out] -1/2*((e*x)^n*(b*Cosh[c + d*x^n])^p*Hypergeometric2F1[1/2, (1 + p)/2, (3 +
p)/2, Cosh[c + d*x^n]^2]*Sinh[2*(c + d*x^n)])/(d*e*n*(1 + p)*x^n*Sqrt[-Sinh
[c + d*x^n]^2])
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$
\int (ex)^{-1+n} (b\cosh(c+d\,x^n))^p \, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x^n + c))^p*(x*e)^(n - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x^n + c))^p*(x*e)^(n - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh (c+dx^n))^p (ex)^{n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(b*cosh(c+d*x**n))**p,x)
```

```
[Out] Integral((b*cosh(c + d*x**n))**p*(e*x)**(n - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {(e\,x)}^{n-1}\,{\left(b\,\mathrm{cosh}\left(c+d\,x^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(n - 1)*(b*cosh(c + d*x^n))^p,x)
```

```
[Out] int((e*x)^(n - 1)*(b*cosh(c + d*x^n))^p, x)
```

3.44 $\int (ex)^{-1+2n} (b\cosh(c+dx^n))^p \, dx$

Optimal. Leaf size=39

$$\frac{x^{-2n}(ex)^{2n}\mathrm{Int}(x^{-1+2n}(b\cosh(c+dx^n))^p, x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(b*cosh(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (b\cosh(c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(b*Cosh[c + d*x^n])^p,x]
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(b*Cosh[c + d*x^n])^p, x])/(e*x^(2*n))
```

Rubi steps

$$\int (ex)^{-1+2n} (b\cosh(c+dx^n))^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n}(b\cosh(c+dx^n))^p \, dx}{e}$$

Mathematica [A]
time = 4.09, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (b\cosh(c+dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(b*Cosh[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Cosh[c + d*x^n])^p, x]
```

Maple [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (b\cosh(c+d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((b*cosh(d*x^n + c))^p*(x*e)^(2*n - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((b*cosh(d*x^n + c))^p*(x*e)^(2*n - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh{(c + dx^n)})^p (ex)^{2n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(b*cosh(c+d*x**n))**p,x)
[Out] Integral((b*cosh(c + d*x**n))**p*(e*x)**(2*n - 1), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c))^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(e\,x\right)}^{2\,n-1}\,{\left(b\,\mathrm{cosh}\left(c+d\,x^n\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(2*n - 1)*(b*cosh(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(b*cosh(c + d*x^n))^p, x)
```

3.45 $\int (ex)^{-1+n} \left(a+b\cosh\left(c+dx^n\right)\right)^p \, dx$

Optimal. Leaf size=131

$$\frac{\sqrt{2}\, x^{-n}(ex)^n F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-\cosh\left(c+dx^n\right)),\frac{b(1-\cosh(c+dx^n))}{a+b}\right)\left(a+b\cosh\left(c+dx^n\right)\right)^p\left(\frac{a+b\cosh(c+dx^n)}{a+b}\right)}{den\sqrt{1+\cosh\left(c+dx^n\right)}}$$

```
[Out] (e*x)^n*AppellF1(1/2,-p,1/2,3/2,b*(1-cosh(c+d*x^n))/(a+b),1/2-1/2*cosh(c+d*
x^n))*(a+b*cosh(c+d*x^n))^p*sinh(c+d*x^n)*2^(1/2)/d/e/n/(x^n)/(((a+b*cosh(c
+d*x^n))/(a+b))^p)/(1+cosh(c+d*x^n))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5431, 5429, 2744, 144, 143}

$$\frac{\sqrt{2}\, x^{-n}(ex)^n \sinh\left(c+dx^n\right)\left(a+b\cosh\left(c+dx^n\right)\right)^p\left(\frac{a+b\cosh(c+dx^n)}{a+b}\right)^{-p} F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-\cosh\left(dx^n+c\right)),\frac{b(1-\cosh(dx^n+c))}{a+b}\right)}{den\sqrt{\cosh\left(c+dx^n\right)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(a + b*Cosh[c + d*x^n])^p,x]
```

```
[Out] (Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Cosh[c + d*x^n])/2, (b*(1
 - Cosh[c + d*x^n]))/(a + b)]*(a + b*Cosh[c + d*x^n])^p*Sinh[c + d*x^n])/(d
*e*n*x^n*Sqrt[1 + Cosh[c + d*x^n]]*((a + b*Cosh[c + d*x^n])/(a + b))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5431

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
tegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (ex)^{-1+n}\left(a+b\cosh\left(c+dx^n\right)\right)^p\,dx=\frac{\left(x^{-n}(ex)^n\right)\int x^{-1+n}\left(a+b\cosh\left(c+dx^n\right)\right)^p\,dx}{e}$$

$$=\frac{\left(x^{-n}(ex)^n\right)\operatorname{Subst}\left(\int (a+b\cosh(c+dx))^p\,dx,x,x^n\right)}{en}$$

$$=-\frac{\left(x^{-n}(ex)^n\sinh\left(c+dx^n\right)\right)\operatorname{Subst}\left(\int\frac{(a+bx)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cosh\left(c\right.\right.}{den\sqrt{1-\cosh\left(c+dx^n\right)}\sqrt{1+\cosh\left(c+dx^n\right)}}$$

$$=-\frac{\left(x^{-n}(ex)^n\left(a+b\cosh\left(c+dx^n\right)\right)^p\left(-\frac{a+b\cosh(c+dx^n)}{-a-b}\right)^{-p}\sinh\left(c+d\right.\right.}{den\sqrt{1-\cosh\left(c+dx^n\right)}\sqrt{1}}$$

$$=\frac{\sqrt{2}\,x^{-n}(ex)^nF_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}\left(1-\cosh\left(c+dx^n\right)\right),\frac{b(1-\cosh(c+dx^n))}{a+b}\right)}{den\sqrt{1+\cosh\left(c+\right.}}$$

Mathematica [A]
time = 0.27, size = 148, normalized size = 1.13

$$\frac{x^{-n}(ex)^nF_1\left(1+p;\frac{1}{2},\frac{1}{2};2+p;\frac{a+b\cosh(c+dx^n)}{a+b},\frac{a+b\cosh(c+dx^n)}{a-b}\right)\sqrt{-\frac{b\left(-1+\cosh\left(c+dx^n\right)\right)}{a+b}}\sqrt{\frac{b\left(1+\cosh\left(c+dx^n\right)\right)}{-a+b}}\left(a+b\cosh\left(c+dx^n\right)\right)^{1+p}\operatorname{csch}(c+dx^n)}{bden(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Cosh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Cosh[c + d*x^n])/(a + b),
(a + b*Cosh[c + d*x^n])/(a - b)]*Sqrt[-((b*(-1 + Cosh[c + d*x^n]))/(a + b))
]*Sqrt[(b*(1 + Cosh[c + d*x^n]))/(-a + b)]*(a + b*Cosh[c + d*x^n])^(1 + p)*
Csch[c + d*x^n])/(b*d*e*n*(1 + p)*x^n)
```

Maple [**F**]
time = 0.83, size = 0, normalized size = 0.00

$$\int (ex)^{-1+n} \left(a + b\cosh\left(c + d\,x^{n}\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x^n + c) + a)^p*(x*e)^(n - 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x^n + c) + a)^p*(x*e)^(n - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(a+b*cosh(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c) + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{n-1}\,(a+b\cosh(c+d\,x^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(n - 1)*(a + b*cosh(c + d*x^n))^p,x)
```

```
[Out] int((e*x)^(n - 1)*(a + b*cosh(c + d*x^n))^p, x)
```

3.46 $\int (ex)^{-1+2n} (a + b\cosh(c + dx^n))^p \, dx$

Optimal. Leaf size=41

$$\frac{x^{-2n}(ex)^{2n}\mathrm{Int}(x^{-1+2n}(a + b\cosh(c + dx^n))^p, x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (a + b\cosh(c + dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Cosh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(a + b*Cosh[c + d*x^n])^p, x])/(e*x^(2
*n))
```

Rubi steps

$$\int (ex)^{-1+2n} (a + b\cosh(c + dx^n))^p \, dx = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(a + b\cosh(c + dx^n))^p \, dx}{e}$$

Mathematica [A]
time = 5.94, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (a + b\cosh(c + dx^n))^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Cosh[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Cosh[c + d*x^n])^p, x]
```

Maple [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (a + b\cosh(c + d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x^n + c) + a)^p*(x*e)^(2*n - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x^n + c) + a)^p*(x*e)^(2*n - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(a+b*cosh(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c) + a)^p, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int (e\,x)^{2\,n-1}\,(a+b\cosh(c+d\,x^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(2*n - 1)*(a + b*cosh(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(a + b*cosh(c + d*x^n))^p, x)
```

3.47 $\int x^m \cosh(a+bx^n)\,dx$

Optimal. Leaf size=89

$$-\frac{e^a x^{1+m}(-bx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-bx^n\right)}{2n}-\frac{e^{-a}x^{1+m}(bx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},bx^n\right)}{2n}$$

```
[Out] -1/2*exp(a)*x^(1+m)*GAMMA((1+m)/n,-b*x^n)/n/((-b*x^n)^((1+m)/n))-1/2*x^(1+m
)*GAMMA((1+m)/n,b*x^n)/exp(a)/n/((b*x^n)^((1+m)/n))
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5469, 2250}

$$-\frac{e^a x^{m+1}(-bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-bx^n\right)}{2n}-\frac{e^{-a}x^{m+1}(bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x^n],x]
```

```
[Out] -1/2*(E^a*x^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(n*(-(b*x^n))^((1 + m)/n))
- (x^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(2*E^a*n*(b*x^n)^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5469

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh(a+bx^n)\,dx &= \frac{1}{2}\int e^{-a-bx^n}x^m\,dx+\frac{1}{2}\int e^{a+bx^n}x^m\,dx\\ &= -\frac{e^a x^{1+m}(-bx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-bx^n\right)}{2n}-\frac{e^{-a}x^{1+m}(bx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},bx^n\right)}{2n}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 100, normalized size = 1.12

$$-\frac{x^{1+m}\left(-b^2x^{2n}\right)^{-\frac{1+m}{n}}\left(\left(-bx^n\right)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},bx^n\right)\left(\cosh(a)-\sinh(a)\right)+\left(bx^n\right)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)\left(\cosh(a)+\sinh(a)\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x^n],x]
```

```
[Out] -1/2*(x^(1 + m)*((-(b*x^n))^((1 + m)/n)*Gamma[(1 + m)/n, b*x^n]*(Cosh[a] -
Sinh[a]) + (b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -(b*x^n)]*(Cosh[a] + Sinh[a
])))/(n*(-(b^2*x^(2*n)))^((1 + m)/n))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.54, size = 110, normalized size = 1.24

method	result
meijerg	$\frac{x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{1+m}+\frac{x^{n+m+1}b\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{n+m+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],1/4*x^(2*n)
*b^2)*cosh(a)+1/(n+m+1)*x^(n+m+1)*b*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,3/2+
1/2/n*m+1/2/n],1/4*x^(2*n)*b^2)*sinh(a)
```

Maxima [A]
time = 0.09, size = 85, normalized size = 0.96

$$-\frac{x^{m+1}e^{(-a)}\Gamma\left(\frac{m+1}{n},bx^n\right)}{2\left(bx^n\right)^{\frac{m+1}{n}}n}-\frac{x^{m+1}e^{a}\Gamma\left(\frac{m+1}{n},-bx^n\right)}{2\left(-bx^n\right)^{\frac{m+1}{n}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/2*x^(m + 1)*e^(-a)*gamma((m + 1)/n, b*x^n)/((b*x^n)^((m + 1)/n)*n) - 1/2
*x^(m + 1)*e^a*gamma((m + 1)/n, -b*x^n)/((-b*x^n)^((m + 1)/n)*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*x**n),x)
```

```
[Out] Integral(x**m*cosh(a + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cosh(a + b\, x^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x^n),x)
```

```
[Out] int(x^m*cosh(a + b*x^n), x)
```

3.48 $\int x^m \cosh^2(a+bx^n)\,dx$

Optimal. Leaf size=128

$$\frac{x^{1+m}}{2(1+m)}-\frac{2^{-\frac{1+m+2n}{n}}e^{2a}x^{1+m}(-bx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-2bx^n\right)}{n}-\frac{2^{-\frac{1+m+2n}{n}}e^{-2a}x^{1+m}(bx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},2bx^n\right)}{n}$$

```
[Out] 1/2*x^(1+m)/(1+m)-exp(2*a)*x^(1+m)*GAMMA((1+m)/n,-2*b*x^n)/(2^((1+m+2*n)/n)
)/n/((-b*x^n)^((1+m)/n))-x^(1+m)*GAMMA((1+m)/n,2*b*x^n)/(2^((1+m+2*n)/n))/e
xp(2*a)/n/((b*x^n)^((1+m)/n))
```

Rubi [A]
time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5471, 5469, 2250}

$$-\frac{e^{2a}2^{-\frac{m+2n+1}{n}}x^{m+1}(-bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{m+2n+1}{n}}x^{m+1}(bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},2bx^n\right)}{n}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x^n]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) - (E^(2*a)*x^(1 + m)*Gamma[(1 + m)/n, -2*b*x^n])/(2^(
(1 + m + 2*n)/n)*n*(-(b*x^n))^((1 + m)/n)) - (x^(1 + m)*Gamma[(1 + m)/n, 2*
b*x^n])/(2^((1 + m + 2*n)/n)*E^(2*a)*n*(b*x^n)^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5469

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5471

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^m \cosh^2(a+bx^n)\,dx = \int \left(\frac{x^m}{2}+\frac{1}{2}x^m\cosh(2a+2bx^n)\right)dx$$
$$= \frac{x^{1+m}}{2(1+m)}+\frac{1}{2}\int x^m\cosh(2a+2bx^n)\,dx$$
$$= \frac{x^{1+m}}{2(1+m)}+\frac{1}{4}\int e^{-2a-2bx^n}x^m\,dx+\frac{1}{4}\int e^{2a+2bx^n}x^m\,dx$$
$$= \frac{x^{1+m}}{2(1+m)}-\frac{2^{-\frac{1+m+2n}{n}}e^{2a}x^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2bx^n\right)}{n}-\frac{2^{-\frac{1+m+2n}{n}}e^{-2a}x^{1+m}(}{}$$

Mathematica [A]
time = 0.18, size = 116, normalized size = 0.91

$$-\frac{x^{1+m}\left(-2n+2^{-\frac{1+m}{n}}e^{2a}(1+m)(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2bx^n\right)+2^{-\frac{1+m}{n}}e^{-2a}(1+m)(bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},2bx^n\right)\right)}{4(1+m)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x^n]^2,x]
```

```
[Out] -1/4*(x^(1 + m)*(-2*n + (E^(2*a)*(1 + m)*Gamma[(1 + m)/n, -2*b*x^n])/(2^((1
 + m)/n)*(-(b*x^n))^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, 2*b*x^n])/(2^(
(1 + m)/n)*E^(2*a)*(b*x^n)^((1 + m)/n))))/((1 + m)*n)
```

Maple [F]
time = 1.11, size = 0, normalized size = 0.00

$$\int x^m\left(\cosh^2(a+b\,x^n)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*x^n)^2,x)
```

```
[Out] int(x^m*cosh(a+b*x^n)^2,x)
```

Maxima [A]
time = 0.10, size = 101, normalized size = 0.79

$$-\frac{x^{m+1}e^{(-2\,a)}\Gamma\left(\frac{m+1}{n},2\,bx^n\right)}{4\,(2\,bx^n)^{\frac{m+1}{n}}\,n}-\frac{x^{m+1}e^{(2\,a)}\Gamma\left(\frac{m+1}{n},-2\,bx^n\right)}{4\,(-2\,bx^n)^{\frac{m+1}{n}}\,n}+\frac{x^{m+1}}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^(m + 1)*e^(-2*a)*gamma((m + 1)/n, 2*b*x^n)/((2*b*x^n)^((m + 1)/n)*n)
 - 1/4*x^(m + 1)*e^(2*a)*gamma((m + 1)/n, -2*b*x^n)/((-2*b*x^n)^((m + 1)/n)
*n) + 1/2*x^(m + 1)/(m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x^n + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh^2(a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*x**n)**2,x)
```

```
[Out] Integral(x**m*cosh(a + b*x**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cosh(a + b\, x^n)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x^n)^2,x)
```

```
[Out] int(x^m*cosh(a + b*x^n)^2, x)
```

3.49 $\int x^m \cosh^3(a+bx^n)\,dx$

Optimal. Leaf size=200

$$-\frac{3^{-\frac{1+m}{n}}e^{3a}x^{1+m}(-bx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-3bx^n\right)}{8n}-\frac{3e^{a}x^{1+m}(-bx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^{1+m}(bx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1}{}\right.}{8n}$$

```
[Out] -1/8*exp(3*a)*x^(1+m)*GAMMA((1+m)/n,-3*b*x^n)/(3^((1+m)/n))/n/((-b*x^n)^((1
+m)/n))-3/8*exp(a)*x^(1+m)*GAMMA((1+m)/n,-b*x^n)/n/((-b*x^n)^((1+m)/n))-3/8
*x^(1+m)*GAMMA((1+m)/n,b*x^n)/exp(a)/n/((b*x^n)^((1+m)/n))-1/8*x^(1+m)*GAMM
A((1+m)/n,3*b*x^n)/(3^((1+m)/n))/exp(3*a)/n/((b*x^n)^((1+m)/n))
```

Rubi [A]
time = 0.14, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5471, 5469, 2250}

$$-\frac{e^{3a}3^{-\frac{m+1}{n}}x^{m+1}(-bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-3bx^n\right)}{8n}-\frac{3e^{a}x^{m+1}(-bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^{m+1}(bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},bx^n\right)}{8n}-\frac{e^{-3a}3^{-\frac{m+1}{n}}x^{m+1}(bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},3bx^n\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(E^(3*a)*x^(1 + m)*Gamma[(1 + m)/n, -3*b*x^n])/(3^((1 + m)/n)*n*(-(b*x
^n))^((1 + m)/n)) - (3*E^a*x^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(8*n*(-(b*
x^n))^((1 + m)/n)) - (3*x^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(8*E^a*n*(b*x^n)
^((1 + m)/n)) - (x^(1 + m)*Gamma[(1 + m)/n, 3*b*x^n])/(8*3^((1 + m)/n)*E^(3
*a)*n*(b*x^n)^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5469

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5471

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^m \cosh^3\left(a+bx^n\right)\,dx &= \int \left(\frac{3}{4}x^m\cosh\left(a+bx^n\right)+\frac{1}{4}x^m\cosh\left(3a+3bx^n\right)\right)\,dx\\
&= \frac{1}{4}\int x^m\cosh\left(3a+3bx^n\right)\,dx+\frac{3}{4}\int x^m\cosh\left(a+bx^n\right)\,dx\\
&= \frac{1}{8}\int e^{-3a-3bx^n}x^m\,dx+\frac{1}{8}\int e^{3a+3bx^n}x^m\,dx+\frac{3}{8}\int e^{-a-bx^n}x^m\,dx+\frac{3}{8}\int e^{a+bx^n}x^m\\
&= -\frac{3^{-\frac{1+m}{n}}e^{3a}x^{1+m}\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3bx^n\right)}{8n}-\frac{3e^{a}x^{1+m}\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-b\right.}{8n}
\end{aligned}
$$

Mathematica [A]
time = 2.66, size = 182, normalized size = 0.91

$$
-\frac{3^{-\frac{1+m}{n}}e^{-3a}x^{1+m}\left(-b^2x^{2n}\right)^{-\frac{1+m}{n}}\left(e^{6a}\left(bx^n\right)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3bx^n\right)+3^{\frac{1+m+n}{n}}e^{4a}\left(bx^n\right)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)+\left(-bx^n\right)^{\frac{1+m}{n}}\left(3^{\frac{1+m+n}{n}}e^{2a}\Gamma\left(\frac{1+m}{n},bx^n\right)+\Gamma\left(\frac{1+m}{n},3bx^n\right)\right)\right)}{8n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(x^(1 + m)*(E^(6*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -3*b*x^n] + 3
^((1 + m + n)/n)*E^(4*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -(b*x^n)] + (
-(b*x^n))^((1 + m)/n)*(3^((1 + m + n)/n)*E^(2*a)*Gamma[(1 + m)/n, b*x^n] +
Gamma[(1 + m)/n, 3*b*x^n])))/(3^((1 + m)/n)*E^(3*a)*n*(-(b^2*x^(2*n)))^((1
+ m)/n))
```

Maple [F]
time = 1.37, size = 0, normalized size = 0.00

$$
\int x^{m}\left(\cosh^{3}\left(a+b\,x^{n}\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*x^n)^3,x)
```

```
[Out] int(x^m*cosh(a+b*x^n)^3,x)
```

Maxima [A]
time = 0.16, size = 173, normalized size = 0.86

$$
-\frac{x^{m+1}e^{(-3\,a)}\Gamma\left(\frac{m+1}{n},3\,bx^{n}\right)}{8\,\left(3\,bx^{n}\right)^{\frac{m+1}{n}}\,n}-\frac{3\,x^{m+1}e^{(-a)}\Gamma\left(\frac{m+1}{n},bx^{n}\right)}{8\,\left(bx^{n}\right)^{\frac{m+1}{n}}\,n}-\frac{3\,x^{m+1}e^{a}\Gamma\left(\frac{m+1}{n},-bx^{n}\right)}{8\,\left(-bx^{n}\right)^{\frac{m+1}{n}}\,n}-\frac{x^{m+1}e^{(3\,a)}\Gamma\left(\frac{m+1}{n},-3\,bx^{n}\right)}{8\,\left(-3\,bx^{n}\right)^{\frac{m+1}{n}}\,n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -1/8*x^(m + 1)*e^(-3*a)*gamma((m + 1)/n, 3*b*x^n)/((3*b*x^n)^((m + 1)/n)*n)
 - 3/8*x^(m + 1)*e^(-a)*gamma((m + 1)/n, b*x^n)/((b*x^n)^((m + 1)/n)*n) - 3
/8*x^(m + 1)*e^a*gamma((m + 1)/n, -b*x^n)/((-b*x^n)^((m + 1)/n)*n) - 1/8*x^
(m + 1)*e^(3*a)*gamma((m + 1)/n, -3*b*x^n)/((-3*b*x^n)^((m + 1)/n)*n)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*cosh(b*x^n + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh^3 (a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*x**n)**3,x)
```

```
[Out] Integral(x**m*cosh(a + b*x**n)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x^n + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \cosh(a + b\, x^n)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x^n)^3,x)
```

```
[Out] int(x^m*cosh(a + b*x^n)^3, x)
```

3.50 $\int x^{-1-n} \cosh(a+bx^n) \, dx$

Optimal. Leaf size=45

$$-\frac{x^{-n}\cosh(a+bx^n)}{n}+\frac{b\mathrm{Chi}(bx^n)\sinh(a)}{n}+\frac{b\cosh(a)\mathrm{Shi}(bx^n)}{n}$$

```
[Out] -cosh(a+b*x^n)/n/(x^n)+b*cosh(a)*Shi(b*x^n)/n+b*Chi(b*x^n)*sinh(a)/n
```

Rubi [A]
time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5429, 3378, 3384, 3379, 3382}

$$\frac{b\sinh(a)\mathrm{Chi}(bx^n)}{n}+\frac{b\cosh(a)\mathrm{Shi}(bx^n)}{n}-\frac{x^{-n}\cosh(a+bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Cosh[a + b*x^n],x]
```

```
[Out] -(Cosh[a + b*x^n]/(n*x^n)) + (b*CoshIntegral[b*x^n]*Sinh[a])/n + (b*Cosh[a]
*SinhIntegral[b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\cosh\left(a+bx^n\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx,x,x^n\right)}{n}\\ &= -\frac{x^{-n}\cosh\left(a+bx^n\right)}{n}+\frac{b\text{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx,x,x^n\right)}{n}\\ &= -\frac{x^{-n}\cosh\left(a+bx^n\right)}{n}+\frac{(b\cosh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx,x,x^n\right)}{n}+\frac{(b\sinh(a))\text{Sub}}{}\\ &= -\frac{x^{-n}\cosh\left(a+bx^n\right)}{n}+\frac{b\text{Chi}(bx^n)\sinh(a)}{n}+\frac{b\cosh(a)\text{Shi}(bx^n)}{n}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 1.02

$$\frac{x^{-n}\left(-\cosh\left(a+bx^n\right)+bx^n\text{Chi}(bx^n)\sinh(a)+bx^n\cosh(a)\text{Shi}(bx^n)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Cosh[a + b*x^n],x]
```

```
[Out] (-Cosh[a + b*x^n] + b*x^n*CoshIntegral[b*x^n]*Sinh[a] + b*x^n*Cosh[a]*SinhI
ntegral[b*x^n])/(n*x^n)
```

Maple [A]
time = 1.44, size = 74, normalized size = 1.64

method	result	size
risch	$-\frac{{\mathrm e}^{-a-b\,x^{n}} x^{-n}}{2 n}+\frac{b\,{\mathrm e}^{-a} \mathrm{expIntegral}(1,b\,x^{n})}{2 n}-\frac{{\mathrm e}^{a+b\,x^{n}} x^{-n}}{2 n}-\frac{b\,{\mathrm e}^{a} \mathrm{expIntegral}(1,-b\,x^{n})}{2 n}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*cosh(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/n*exp(-a-b*x^n)/(x^n)+1/2/n*b*exp(-a)*Ei(1,b*x^n)-1/2*exp(a+b*x^n)/(x^
n)/n-1/2/n*b*exp(a)*Ei(1,-b*x^n)
```

Maxima [A]
time = 0.32, size = 34, normalized size = 0.76

$$-\frac{be^{(-a)}\Gamma(-1,bx^n)}{2n}+\frac{be^{a}\Gamma(-1,-bx^n)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/2*b*e^(-a)*gamma(-1, b*x^n)/n + 1/2*b*e^a*gamma(-1, -b*x^n)/n
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(45) = 90$.
time = 0.40, size = 140, normalized size = 3.11

$$\frac{((b\cosh(a)+b\sinh(a))\cosh(n\log(x))+(b\cosh(a)+b\sinh(a))\sinh(n\log(x)))\operatorname{Ei}(b\cosh(n\log(x))+b\sinh(n\log(x)))-((b\cosh(a)-b\sinh(a))\cosh(n\log(x))+(b\cosh(a)-b\sinh(a))\sinh(n\log(x)))\operatorname{Ei}(-b\cosh(n\log(x))-b\sinh(n\log(x)))-2\cosh(b\cosh(n\log(x))+b\sinh(n\log(x))+a)}{2(n\cosh(n\log(x))+n\sinh(n\log(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh
(n*log(x)))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) - ((b*cosh(a) - b*sinh(
a))*cosh(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*l
og(x)) - b*sinh(n*log(x))) - 2*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a
))/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\cosh\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*cosh(a+b*x**n),x)
```

```
[Out] Integral(x**(-n - 1)*cosh(a + b*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*cosh(b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(a + b\,x^n)}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^n)/x^(n + 1),x)
```

```
[Out] int(cosh(a + b*x^n)/x^(n + 1), x)
```

3.51 $\int x^{-1-n}\cosh^2(a+bx^n)\,dx$

Optimal. Leaf size=67

$$-\frac{x^{-n}}{2n}-\frac{x^{-n}\cosh\left(2(a+bx^n)\right)}{2n}+\frac{b\text{Chi}(2bx^n)\sinh(2a)}{n}+\frac{b\cosh(2a)\text{Shi}(2bx^n)}{n}$$

```
[Out] -1/2/n/(x^n)-1/2*cosh(2*a+2*b*x^n)/n/(x^n)+b*cosh(2*a)*Shi(2*b*x^n)/n+b*Chi
(2*b*x^n)*sinh(2*a)/n
```

Rubi [A]
time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {5471, 5429, 3378, 3384, 3379, 3382}

$$\frac{b\sinh(2a)\text{Chi}(2bx^n)}{n}+\frac{b\cosh(2a)\text{Shi}(2bx^n)}{n}-\frac{x^{-n}\cosh\left(2(a+bx^n)\right)}{2n}-\frac{x^{-n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Cosh[a + b*x^n]^2,x]
```

```
[Out] -1/2*1/(n*x^n) - Cosh[2*(a + b*x^n)]/(2*n*x^n) + (b*CoshIntegral[2*b*x^n]*S
inh[2*a])/n + (b*Cosh[2*a]*SinhIntegral[2*b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5471

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-n}\cosh^2\left(a+bx^n\right)\,dx &= \int \left(\frac{x^{-1-n}}{2}+\frac{1}{2}x^{-1-n}\cosh\left(2a+2bx^n\right)\right)\,dx\\
&= -\frac{x^{-n}}{2n}+\frac{1}{2}\int x^{-1-n}\cosh\left(2a+2bx^n\right)\,dx\\
&= -\frac{x^{-n}}{2n}+\frac{\operatorname{Subst}\left(\int \frac{\cosh(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n}\\
&= -\frac{x^{-n}}{2n}-\frac{x^{-n}\cosh\left(2(a+bx^n)\right)}{2n}+\frac{b\operatorname{Subst}\left(\int \frac{\sinh(2a+2bx)}{x}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-n}}{2n}-\frac{x^{-n}\cosh\left(2(a+bx^n)\right)}{2n}+\frac{(b\cosh(2a))\operatorname{Subst}\left(\int \frac{\sinh(2bx)}{x}\,dx,x,x^n\right)}{n}+\frac{(b
\\
&= -\frac{x^{-n}}{2n}-\frac{x^{-n}\cosh\left(2(a+bx^n)\right)}{2n}+\frac{b\operatorname{Chi}(2bx^n)\sinh(2a)}{n}+\frac{b\cosh(2a)\operatorname{Shi}(2bx^n)}{n}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.81

$$\frac{x^{-n}\left(-\cosh^2\left(a+bx^n\right)+bx^n\operatorname{Chi}(2bx^n)\sinh(2a)+bx^n\cosh(2a)\operatorname{Shi}(2bx^n)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Cosh[a + b*x^n]^2,x]
```

```
[Out] (-Cosh[a + b*x^n]^2 + b*x^n*CoshIntegral[2*b*x^n]*Sinh[2*a] + b*x^n*Cosh[2*
a]*SinhIntegral[2*b*x^n])/(n*x^n)
```

Maple [A]
time = 3.00, size = 90, normalized size = 1.34

method	result	size
risch	$-\frac{x^{-n}}{2n}-\frac{\mathrm{e}^{-2a-2b\,x^{n}}x^{-n}}{4n}+\frac{b\,\mathrm{e}^{-2a}\,\mathrm{expIntegral}(1,2b\,x^{n})}{2n}-\frac{x^{-n}\mathrm{e}^{2a+2b\,x^{n}}}{4n}-\frac{b\,\mathrm{e}^{2a}\,\mathrm{expIntegral}(1,-2b\,x^{n})}{2n}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*cosh(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/n/(x^n)-1/4/n*exp(-2*a-2*b*x^n)/(x^n)+1/2/n*b*exp(-2*a)*Ei(1,2*b*x^n)-
1/4/(x^n)*exp(2*a+2*b*x^n)/n-1/2/n*b*exp(2*a)*Ei(1,-2*b*x^n)
```

Maxima [A]
time = 0.34, size = 47, normalized size = 0.70

$$-\frac{be^{(-2\,a)}\Gamma\left(-1,2\,bx^{n}\right)}{2\,n}+\frac{be^{(2\,a)}\Gamma\left(-1,-2\,bx^{n}\right)}{2\,n}-\frac{1}{2\,nx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*e^(-2*a)*gamma(-1, 2*b*x^n)/n + 1/2*b*e^(2*a)*gamma(-1, -2*b*x^n)/n
- 1/2/(n*x^n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(64) = 128.
time = 0.38, size = 182, normalized size = 2.72

$$\frac{\left(\left(b\cosh\left(2\,a\right)+b\sinh\left(2\,a\right)\right)\cosh\left(n\log\left(x\right)\right)+\left(b\cosh\left(2\,a\right)+b\sinh\left(2\,a\right)\right)\sinh\left(n\log\left(x\right)\right)\right){\rm Ei}\left(2\,b\cosh\left(n\log\left(x\right)\right)+2\,b\sinh\left(n\log\left(x\right)\right)\right)-\left(\left(b\cosh\left(2\,a\right)-b\sinh\left(2\,a\right)\right)\cosh\left(n\log\left(x\right)\right)+\left(b\cosh\left(2\,a\right)-b\sinh\left(2\,a\right)\right)\sinh\left(n\log\left(x\right)\right)\right){\rm Ei}\left(-2\,b\cosh\left(n\log\left(x\right)\right)-2\,b\sinh\left(n\log\left(x\right)\right)\right)-\cosh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)^{2}-\sinh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)^{2}-1}{2\left(n\cosh\left(n\log\left(x\right)\right)+n\sinh\left(n\log\left(x\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((b*cosh(2*a) + b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) + b*sinh(2*
a))*sinh(n*log(x)))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) - ((b*cosh(
2*a) - b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) - b*sinh(2*a))*sinh(n*log
(x)))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) - cosh(b*cosh(n*log(x))
+ b*sinh(n*log(x)) + a)^2 - sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^2
 - 1)/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\cosh^{2}\left(a+bx^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*cosh(a+b*x**n)**2,x)
```

```
[Out] Integral(x**(-n - 1)*cosh(a + b*x**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*cosh(b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x^n)^2}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^n)^2/x^(n + 1),x)
```

```
[Out] int(cosh(a + b*x^n)^2/x^(n + 1), x)
```

3.52 $\int x^{-1-n} \cosh^3(a+bx^n)\, dx$

Optimal. Leaf size=113

$$-\frac{3x^{-n}\cosh(a+bx^n)}{4n}-\frac{x^{-n}\cosh(3(a+bx^n))}{4n}+\frac{3b\mathrm{Chi}(bx^n)\sinh(a)}{4n}+\frac{3b\mathrm{Chi}(3bx^n)\sinh(3a)}{4n}+\frac{3b\cosh(a)\mathrm{Shi}(}{4n}$$

```
[Out] -3/4*cosh(a+b*x^n)/n/(x^n)-1/4*cosh(3*a+3*b*x^n)/n/(x^n)+3/4*b*cosh(a)*Shi(
b*x^n)/n+3/4*b*cosh(3*a)*Shi(3*b*x^n)/n+3/4*b*Chi(b*x^n)*sinh(a)/n+3/4*b*Ch
i(3*b*x^n)*sinh(3*a)/n
```

Rubi [A]
time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5471, 5429, 3378, 3384, 3379, 3382}

$$\frac{3b\sinh(a)\mathrm{Chi}(bx^n)}{4n}+\frac{3b\sinh(3a)\mathrm{Chi}(3bx^n)}{4n}+\frac{3b\cosh(a)\mathrm{Shi}(bx^n)}{4n}+\frac{3b\cosh(3a)\mathrm{Shi}(3bx^n)}{4n}-\frac{3x^{-n}\cosh(a+bx^n)}{4n}-\frac{x^{-n}\cosh(3(a+bx^n))}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Cosh[a + b*x^n]^3,x]
```

```
[Out] (-3*Cosh[a + b*x^n])/(4*n*x^n) - Cosh[3*(a + b*x^n)]/(4*n*x^n) + (3*b*CoshI
ntegral[b*x^n]*Sinh[a])/(4*n) + (3*b*CoshIntegral[3*b*x^n]*Sinh[3*a])/(4*n)
 + (3*b*Cosh[a]*SinhIntegral[b*x^n])/(4*n) + (3*b*Cosh[3*a]*SinhIntegral[3*
b*x^n])/(4*n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5471

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-n}\cosh^3\left(a+bx^n\right)\,dx &= \int \left(\frac{3}{4}x^{-1-n}\cosh\left(a+bx^n\right)+\frac{1}{4}x^{-1-n}\cosh\left(3a+3bx^n\right)\right)\,dx\\
&= \frac{1}{4}\int x^{-1-n}\cosh\left(3a+3bx^n\right)\,dx+\frac{3}{4}\int x^{-1-n}\cosh\left(a+bx^n\right)\,dx\\
&= \frac{\mathrm{Subst}\left(\int \frac{\cosh(3a+3bx)}{x^2}\,dx,x,x^n\right)}{4n}+\frac{3\mathrm{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx,x,x^n\right)}{4n}\\
&= -\frac{3x^{-n}\cosh\left(a+bx^n\right)}{4n}-\frac{x^{-n}\cosh\left(3(a+bx^n)\right)}{4n}+\frac{(3b)\mathrm{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx,x\right.}{4n}\\
&= -\frac{3x^{-n}\cosh\left(a+bx^n\right)}{4n}-\frac{x^{-n}\cosh\left(3(a+bx^n)\right)}{4n}+\frac{(3b\cosh(a))\mathrm{Subst}\left(\int \frac{\sinh(bx)}{x}\right.}{4n}\\
&= -\frac{3x^{-n}\cosh\left(a+bx^n\right)}{4n}-\frac{x^{-n}\cosh\left(3(a+bx^n)\right)}{4n}+\frac{3b\mathrm{Chi}(bx^n)\sinh(a)}{4n}+\frac{3b\mathrm{Ch}}{}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 97, normalized size = 0.86

$$\frac{x^{-n}\left(-3\cosh\left(a+bx^n\right)-\cosh\left(3(a+bx^n)\right)+3bx^n\mathrm{Chi}(bx^n)\sinh(a)+3bx^n\mathrm{Chi}(3bx^n)\sinh(3a)+3bx^n\cosh(a)\mathrm{Shi}(bx^n)+3bx^n\cosh(3a)\mathrm{Shi}(3bx^n)\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Cosh[a + b*x^n]^3,x]
```

```
[Out] (-3*Cosh[a + b*x^n] - Cosh[3*(a + b*x^n)] + 3*b*x^n*CoshIntegral[b*x^n]*Sin
h[a] + 3*b*x^n*CoshIntegral[3*b*x^n]*Sinh[3*a] + 3*b*x^n*Cosh[a]*SinhIntegr
al[b*x^n] + 3*b*x^n*Cosh[3*a]*SinhIntegral[3*b*x^n])/(4*n*x^n)
```

Maple [A]
time = 3.55, size = 152, normalized size = 1.35

method	result
risch	$-\frac{e^{-3a-3b\,x^{n}}x^{-n}}{8n}+\frac{3b\,e^{-3a}\,\mathrm{expIntegral}(1,3b\,x^{n})}{8n}-\frac{3\,e^{-a-b\,x^{n}}x^{-n}}{8n}+\frac{3b\,e^{-a}\,\mathrm{expIntegral}(1,b\,x^{n})}{8n}-\frac{3\,e^{a+b\,x^{n}}x^{-n}}{8n}-\frac{3b\,e^{a}\,\mathrm{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*cosh(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/n*exp(-3*a-3*b*x^n)/(x^n)+3/8/n*b*exp(-3*a)*Ei(1,3*b*x^n)-3/8/n*exp(-a
-b*x^n)/(x^n)+3/8/n*b*exp(-a)*Ei(1,b*x^n)-3/8*exp(a+b*x^n)/(x^n)/n-3/8/n*b*
exp(a)*Ei(1,-b*x^n)-1/8/(x^n)*exp(3*a+3*b*x^n)/n-3/8/n*b*exp(3*a)*Ei(1,-3*b
*x^n)
```

Maxima [A]
time = 0.35, size = 70, normalized size = 0.62

$$-\frac{3\,be^{(-3\,a)}\Gamma(-1,3\,bx^{n})}{8\,n}-\frac{3\,be^{(-a)}\Gamma(-1,bx^{n})}{8\,n}+\frac{3\,be^{a}\Gamma(-1,-bx^{n})}{8\,n}+\frac{3\,be^{(3\,a)}\Gamma(-1,-3\,bx^{n})}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -3/8*b*e^(-3*a)*gamma(-1, 3*b*x^n)/n - 3/8*b*e^(-a)*gamma(-1, b*x^n)/n + 3/
8*b*e^a*gamma(-1, -b*x^n)/n + 3/8*b*e^(3*a)*gamma(-1, -3*b*x^n)/n
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(102) = 204.
time = 0.37, size = 320, normalized size = 2.83

$$-\frac{2\cosh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)^{3}+6\cosh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)\sinh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)^{2}-3\left(\left(b\cosh\left(3a\right)+b\sinh\left(3a\right)\right)\cosh\left(n\log\left(x\right)\right)+\left(b\cosh\left(3a\right)+b\sinh\left(3a\right)\right)\sinh\left(n\log\left(x\right)\right)\right)\mathrm{Ei}\left(3b\cosh\left(n\log\left(x\right)\right)+3b\sinh\left(n\log\left(x\right)\right)\right)-3\left(\left(b\cosh\left(a\right)+b\sinh\left(a\right)\right)\cosh\left(n\log\left(x\right)\right)+\left(b\cosh\left(a\right)+b\sinh\left(a\right)\right)\sinh\left(n\log\left(x\right)\right)\right)\mathrm{Ei}\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)\right)+3\left(\left(b\cosh\left(a\right)-b\sinh\left(a\right)\right)\cosh\left(n\log\left(x\right)\right)+\left(b\cosh\left(a\right)-b\sinh\left(a\right)\right)\sinh\left(n\log\left(x\right)\right)\right)\mathrm{Ei}\left(-b\cosh\left(n\log\left(x\right)\right)-b\sinh\left(n\log\left(x\right)\right)\right)+3\left(\left(b\cosh\left(3a\right)-b\sinh\left(3a\right)\right)\cosh\left(n\log\left(x\right)\right)+\left(b\cosh\left(3a\right)-b\sinh\left(3a\right)\right)\sinh\left(n\log\left(x\right)\right)\right)\mathrm{Ei}\left(-3b\cosh\left(n\log\left(x\right)\right)-3b\sinh\left(n\log\left(x\right)\right)\right)+6\cosh\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)}{8\left(n\cosh\left(n\log\left(x\right)\right)+n\sinh\left(n\log\left(x\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^3 + 6*cosh(b*cosh(n*l
og(x)) + b*sinh(n*log(x)) + a)*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a
)^2 - 3*((b*cosh(3*a) + b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) + b*sinh
(3*a))*sinh(n*log(x)))*Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) - 3*((b*
cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh(n*log(x)
```

```
))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + 3*((b*cosh(a) - b*sinh(a))*cos
h(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*log(x))
- b*sinh(n*log(x))) + 3*((b*cosh(3*a) - b*sinh(3*a))*cosh(n*log(x)) + (b*co
sh(3*a) - b*sinh(3*a))*sinh(n*log(x)))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*
log(x))) + 6*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a))/(n*cosh(n*log(x
)) + n*sinh(n*log(x)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*cosh(a+b*x**n)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*cosh(b*x^n + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x^n)^3}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x^n)^3/x^(n + 1),x)
```

```
[Out] int(cosh(a + b*x^n)^3/x^(n + 1), x)
```

3.53 $\int x^{-1+\frac{n}{2}} \cosh(a+bx^n)\, dx$

Optimal. Leaf size=71

$$\frac{e^{-a}\sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{b}\,x^{n/2}\right)}{2\sqrt{b}\,n}+\frac{e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{b}\,x^{n/2}\right)}{2\sqrt{b}\,n}$$

```
[Out] 1/2*erf(x^(1/2*n)*b^(1/2))*Pi^(1/2)/exp(a)/n/b^(1/2)+1/2*exp(a)*erfi(x^(1/2
*n)*b^(1/2))*Pi^(1/2)/n/b^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5465, 5407, 2235, 2236}

$$\frac{\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\sqrt{b}\,x^{n/2}\right)}{2\sqrt{b}\,n}+\frac{\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{b}\,x^{n/2}\right)}{2\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)*Cosh[a + b*x^n],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[b]*x^(n/2)])/(2*Sqrt[b]*E^a*n) + (E^a*Sqrt[Pi]*Erfi[Sqrt
[b]*x^(n/2)])/(2*Sqrt[b]*n)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5465

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/(m + 1), Subst[Int[(a + b*Cosh[c + d*x^Simplify[n/(m + 1)]])^p
```

```
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegerQ[p] && N
eQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int x^{-1+\frac{n}{2}} \cosh\left(a+bx^n\right) dx &= \frac{2\text{Subst}\left(\int \cosh\left(a+bx^2\right) dx, x, x^{n/2}\right)}{n} \\
&= \frac{\text{Subst}\left(\int e^{-a-bx^2} dx, x, x^{n/2}\right)}{n} + \frac{\text{Subst}\left(\int e^{a+bx^2} dx, x, x^{n/2}\right)}{n} \\
&= \frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\sqrt{b}\,x^{n/2}\right)}{2\sqrt{b}\,n} + \frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}\,x^{n/2}\right)}{2\sqrt{b}\,n}
\end{aligned}
$$

Mathematica [**A**]
time = 0.53, size = 60, normalized size = 0.85

$$
\frac{\sqrt{\pi}\left(\text{Erf}\left(\sqrt{b}\,x^{n/2}\right)(\cosh(a)-\sinh(a))+\text{Erfi}\left(\sqrt{b}\,x^{n/2}\right)(\cosh(a)+\sinh(a))\right)}{2\sqrt{b}\,n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)*Cosh[a + b*x^n],x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]*x^(n/2)]*(Cosh[a] - Sinh[a]) + Erfi[Sqrt[b]*x^(n/2)]
*(Cosh[a] + Sinh[a])))/(2*Sqrt[b]*n)
```

Maple [**A**]
time = 0.48, size = 54, normalized size = 0.76

method	result
risch	$\frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2n\sqrt{b}}+\frac{e^{a}\sqrt{\pi}\,\text{erf}\left(\sqrt{-b}\,x^{\frac{n}{2}}\right)}{2n\sqrt{-b}}$
meijerg	$\frac{\sqrt{2}\,\sqrt{\pi}\left(\frac{\sqrt{ib}\,\sqrt{2}\,\text{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2\sqrt{b}}+\frac{\sqrt{ib}\,\sqrt{2}\,\text{erfi}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2\sqrt{b}}\right)\cosh(a)}{2\sqrt{ib}\,n}-\frac{i\sqrt{2}\,\sqrt{\pi}\left(-\frac{\sqrt{2}\,(ib)^{\frac{3}{2}}\,\text{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2b^{\frac{3}{2}}}+\right.}{2\sqrt{ib}\,n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)*cosh(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/n*exp(-a)*Pi^(1/2)/b^(1/2)*erf(x^(1/2*n)*b^(1/2))+1/2/n*exp(a)*Pi^(1/2)
/(-b)^(1/2)*erf((-b)^(1/2)*x^(1/2*n))
```

Maxima [A]
time = 0.32, size = 69, normalized size = 0.97

$$\frac{\sqrt{\pi}\, x^{\frac{1}{2}\,n} \left(\operatorname{erf}\left(\sqrt{bx^n}\right) - 1\right) e^{(-a)}}{2\,\sqrt{bx^n}\,n} + \frac{\sqrt{\pi}\, x^{\frac{1}{2}\,n} \left(\operatorname{erf}\left(\sqrt{-bx^n}\right) - 1\right) e^{a}}{2\,\sqrt{-bx^n}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)*cosh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(pi)*x^(1/2*n)*(erf(sqrt(b*x^n)) - 1)*e^(-a)/(sqrt(b*x^n)*n) + 1/2*
sqrt(pi)*x^(1/2*n)*(erf(sqrt(-b*x^n)) - 1)*e^a/(sqrt(-b*x^n)*n)
```

Fricas [A]
time = 0.36, size = 98, normalized size = 1.38

$$-\frac{\sqrt{\pi}\,\sqrt{-b}\,(\cosh(a)+\sinh(a))\operatorname{erf}\left(\sqrt{-b}\,x\cosh\left(\frac{1}{2}\,(n-2)\log(x)\right)+\sqrt{-b}\,x\sinh\left(\frac{1}{2}\,(n-2)\log(x)\right)\right)-\sqrt{\pi}\,\sqrt{b}\,(\cosh(a)-\sinh(a))\operatorname{erf}\left(\sqrt{b}\,x\cosh\left(\frac{1}{2}\,(n-2)\log(x)\right)+\sqrt{b}\,x\sinh\left(\frac{1}{2}\,(n-2)\log(x)\right)\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)*cosh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x*cosh(1/2*(n - 2)
*log(x)) + sqrt(-b)*x*sinh(1/2*(n - 2)*log(x))) - sqrt(pi)*sqrt(b)*(cosh(a)
 - sinh(a))*erf(sqrt(b)*x*cosh(1/2*(n - 2)*log(x)) + sqrt(b)*x*sinh(1/2*(n
- 2)*log(x))))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{n}{2}-1}\cosh\left(a+bx^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)*cosh(a+b*x**n),x)
```

```
[Out] Integral(x**(n/2 - 1)*cosh(a + b*x**n), x)
```

Giac [A]
time = 0.41, size = 52, normalized size = 0.73

$$-\frac{\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{b}\,\sqrt{x^n}\right)e^{(-a)}}{\sqrt{b}} + \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-b}\,\sqrt{x^n}\right)e^{a}}{\sqrt{-b}}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)*cosh(a+b*x^n),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(pi)*erf(-sqrt(b)*sqrt(x^n))*e^(-a)/sqrt(b) + sqrt(pi)*erf(-sqrt(
-b)*sqrt(x^n))*e^a/sqrt(-b))/n
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{\frac{n}{2}-1}\cosh(a+b\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)*cosh(a + b*x^n),x)
```

```
[Out] int(x^(n/2 - 1)*cosh(a + b*x^n), x)
```

3.54 $\int x^2 \cosh \left((a+bx)^2\right) \, dx$

Optimal. Leaf size=113

$$\frac{\sqrt{\pi}\,\mathrm{Erf}(a+bx)}{8b^3}+\frac{a^2\sqrt{\pi}\,\mathrm{Erf}(a+bx)}{4b^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}(a+bx)}{8b^3}+\frac{a^2\sqrt{\pi}\,\mathrm{Erfi}(a+bx)}{4b^3}-\frac{a\sinh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)}{}$$

```
[Out] -a*sinh((b*x+a)^2)/b^3+1/2*(b*x+a)*sinh((b*x+a)^2)/b^3+1/8*erf(b*x+a)*Pi^(1
/2)/b^3+1/4*a^2*erf(b*x+a)*Pi^(1/2)/b^3-1/8*erfi(b*x+a)*Pi^(1/2)/b^3+1/4*a^
2*erfi(b*x+a)*Pi^(1/2)/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5473, 6874, 5407, 2235, 2236, 5429, 2717, 5433, 5406}

$$\frac{\sqrt{\pi}\,a^2\mathrm{Erf}(a+bx)}{4b^3}+\frac{\sqrt{\pi}\,a^2\mathrm{Erfi}(a+bx)}{4b^3}+\frac{\sqrt{\pi}\,\mathrm{Erf}(a+bx)}{8b^3}-\frac{\sqrt{\pi}\,\mathrm{Erfi}(a+bx)}{8b^3}-\frac{a\sinh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\sinh\left((a+bx)^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[(a + b*x)^2],x]
```

```
[Out] (Sqrt[Pi]*Erf[a + b*x])/(8*b^3) + (a^2*Sqrt[Pi]*Erf[a + b*x])/(4*b^3) - (Sq
rt[Pi]*Erfi[a + b*x])/(8*b^3) + (a^2*Sqrt[Pi]*Erfi[a + b*x])/(4*b^3) - (a*S
inh[(a + b*x)^2])/b^3 + ((a + b*x)*Sinh[(a + b*x)^2])/(2*b^3)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
```

```
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \cosh\left((a+bx)^2\right) dx = \frac{\text{Subst}\left(\int (-a+x)^2 \cosh\left(x^2\right) dx, x, a+bx\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int \left(a^2 \cosh\left(x^2\right) - 2ax \cosh\left(x^2\right) + x^2 \cosh\left(x^2\right)\right) dx, x, a+bx\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int x^2 \cosh\left(x^2\right) dx, x, a+bx\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int x \cosh\left(x^2\right) dx, x, a+bx\right)}{b^3} +$$
$$= \frac{(a+bx)\sinh\left((a+bx)^2\right)}{2b^3} - \frac{\text{Subst}\left(\int \sinh\left(x^2\right) dx, x, a+bx\right)}{2b^3} - \frac{a\text{Subst}\left(\int \cosh(\right.}{}$$
$$= \frac{a^2\sqrt{\pi}\,\text{erf}(a+bx)}{4b^3} + \frac{a^2\sqrt{\pi}\,\text{erfi}(a+bx)}{4b^3} - \frac{a\sinh\left((a+bx)^2\right)}{b^3} + \frac{(a+bx)\sinh\left((\right.}{2b^3}$$
$$= \frac{\sqrt{\pi}\,\text{erf}(a+bx)}{8b^3} + \frac{a^2\sqrt{\pi}\,\text{erf}(a+bx)}{4b^3} - \frac{\sqrt{\pi}\,\text{erfi}(a+bx)}{8b^3} + \frac{a^2\sqrt{\pi}\,\text{erfi}(a+bx)}{4b^3}$$

Mathematica [**A**]
time = 0.09, size = 62, normalized size = 0.55

$$\frac{(1+2a^2)\sqrt{\pi}\,\text{Erf}(a+bx) + (-1+2a^2)\sqrt{\pi}\,\text{Erfi}(a+bx) - 4(a-bx)\sinh\left((a+bx)^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[(a + b*x)^2],x]
```

```
[Out] ((1 + 2*a^2)*Sqrt[Pi]*Erf[a + b*x] + (-1 + 2*a^2)*Sqrt[Pi]*Erfi[a + b*x] -
4*(a - b*x)*Sinh[(a + b*x)^2])/(8*b^3)
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.80, size = 136, normalized size = 1.20

method	result
risch	$-\frac{x\,e^{-(bx+a)^2}}{4b^2} + \frac{a\,e^{-(bx+a)^2}}{4b^3} + \frac{a^2\,\text{erf}(bx+a)\sqrt{\pi}}{4b^3} + \frac{\text{erf}(bx+a)\sqrt{\pi}}{8b^3} + \frac{x\,e^{(bx+a)^2}}{4b^2} - \frac{a\,e^{(bx+a)^2}}{4b^3} - \frac{ia^2\sqrt{\pi}\;\text{erf}(ibx+ia)}{4b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh((b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^2*x*exp(-(b*x+a)^2)+1/4*a/b^3*exp(-(b*x+a)^2)+1/4*a^2*erf(b*x+a)*Pi^
(1/2)/b^3+1/8*erf(b*x+a)*Pi^(1/2)/b^3+1/4/b^2*x*exp((b*x+a)^2)-1/4*a/b^3*ex
p((b*x+a)^2)-1/4*I*a^2/b^3*Pi^(1/2)*erf(I*b*x+I*a)+1/8*I/b^3*Pi^(1/2)*erf(I
*b*x+I*a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(95) = 190$.

time = 0.50, size = 818, normalized size = 7.24

$$\frac{1}{3}x^3\cosh\left((bx+a)^2\right) - \frac{1}{6}\left(\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^3b^4\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}(-b^2)^{\frac{7}{2}}} - \frac{3(b^2x+ab)^3ab^4\Gamma\left(\frac{3}{2},\frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{3}{2}}(-b^2)^{\frac{7}{2}}} + \frac{3a^2b^4e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{7}{2}}} + \frac{b^4\Gamma\left(2,\frac{(b^2x+ab)^2}{b^2}\right)}{(-b^2)^{\frac{7}{2}}}\right)a}{\sqrt{-b^2}} + \frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^4b^5\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}(-b^2)^{\frac{9}{2}}} - \frac{6(b^2x+ab)^3a^2b^5\Gamma\left(\frac{3}{2},\frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{3}{2}}(-b^2)^{\frac{9}{2}}} + \frac{4a^3b^5e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{9}{2}}} - \frac{(b^2x+ab)^5b^5\Gamma\left(\frac{5}{2},\frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{5}{2}}(-b^2)^{\frac{9}{2}}} + \frac{4ab^5\Gamma\left(2,\frac{(b^2x+ab)^2}{b^2}\right)}{(-b^2)^{\frac{9}{2}}}\right)b}{\sqrt{-b^2}} - \frac{a\left(\frac{\sqrt{\pi}(b^2x+ab)a^3\left(\operatorname{erf}\left(\sqrt{-\frac{(b^2x+ab)^2}{b^2}}\right)-1\right)}{b^4\sqrt{-\frac{(b^2x+ab)^2}{b^2}}} - \frac{3a^2e^{\left(\frac{(b^2x+ab)^2}{b^2}\right)}}{b^3} + \frac{\Gamma\left(2,-\frac{(b^2x+ab)^2}{b^2}\right)}{b^3} - \frac{3(b^2x+ab)^3a\Gamma\left(\frac{3}{2},-\frac{(b^2x+ab)^2}{b^2}\right)}{b^6\left(-\frac{(b^2x+ab)^2}{b^2}\right)^{\frac{3}{2}}}\right)}{b} + \left(\frac{\sqrt{\pi}(b^2x+ab)a^4\left(\operatorname{erf}\left(\sqrt{-\frac{(b^2x+ab)^2}{b^2}}\right)-1\right)}{b^5\sqrt{-\frac{(b^2x+ab)^2}{b^2}}} - \frac{4a^3e^{\left(\frac{(b^2x+ab)^2}{b^2}\right)}}{b^4} + \frac{4a\Gamma\left(2,-\frac{(b^2x+ab)^2}{b^2}\right)}{b^4} - \frac{6(b^2x+ab)^3a^2\Gamma\left(\frac{3}{2},-\frac{(b^2x+ab)^2}{b^2}\right)}{b^7\left(-\frac{(b^2x+ab)^2}{b^2}\right)^{\frac{3}{2}}} - \frac{(b^2x+ab)^5\Gamma\left(\frac{5}{2},-\frac{(b^2x+ab)^2}{b^2}\right)}{b^9\left(-\frac{(b^2x+ab)^2}{b^2}\right)^{\frac{5}{2}}}\right)b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*cosh((b*x + a)^2) - 1/6*((sqrt(pi)*(b^2*x + a*b)*a^3*b^4*(erf(sqrt(
(b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(7/2)) - 3*(b^2*x +
a*b)^3*a*b^4*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2
)^(7/2)) + 3*a^2*b^4*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(7/2) + b^4*gamma(2, (
b^2*x + a*b)^2/b^2)/(-b^2)^(7/2))*a/sqrt(-b^2) + (sqrt(pi)*(b^2*x + a*b)*a^
4*b^5*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(9/2
)) - 6*(b^2*x + a*b)^3*a^2*b^5*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a
*b)^2)^(3/2)*(-b^2)^(9/2)) + 4*a^3*b^5*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(9/2
) - (b^2*x + a*b)^5*b^5*gamma(5/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^
(5/2)*(-b^2)^(9/2)) + 4*a*b^5*gamma(2, (b^2*x + a*b)^2/b^2)/(-b^2)^(9/2))*b
/sqrt(-b^2) - a*(sqrt(pi)*(b^2*x + a*b)*a^3*(erf(sqrt(-(b^2*x + a*b)^2/b^2)
) - 1)/(b^4*sqrt(-(b^2*x + a*b)^2/b^2)) - 3*a^2*e^((b^2*x + a*b)^2/b^2)/b^3
 + gamma(2, -(b^2*x + a*b)^2/b^2)/b^3 - 3*(b^2*x + a*b)^3*a*gamma(3/2, -(b^
2*x + a*b)^2/b^2)/(b^6*(-(b^2*x + a*b)^2/b^2)^(3/2)))/b + sqrt(pi)*(b^2*x +
 a*b)*a^4*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/(b^5*sqrt(-(b^2*x + a*b)^2/
b^2)) - 4*a^3*e^((b^2*x + a*b)^2/b^2)/b^4 + 4*a*gamma(2, -(b^2*x + a*b)^2/b
^2)/b^4 - 6*(b^2*x + a*b)^3*a^2*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^7*(-(b^
2*x + a*b)^2/b^2)^(3/2)) - (b^2*x + a*b)^5*gamma(5/2, -(b^2*x + a*b)^2/b^2)
/(b^9*(-(b^2*x + a*b)^2/b^2)^(5/2)))*b
```

Fricas [A]

time = 0.50, size = 164, normalized size = 1.45

$$\frac{\left(\sqrt{\pi}\,(2\,a^2+1)\sqrt{b^2}\,\operatorname{erf}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)e^{(b^2x^2+2\,abx+a^2)} + \sqrt{\pi}\,(2\,a^2-1)\sqrt{b^2}\,\operatorname{erfi}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)e^{(b^2x^2+2\,abx+a^2)} - 2\,b^2x + 2\,ab + 2\,(b^2x-ab)e^{(2\,b^2x^2+4\,abx+2\,a^2)}\right)e^{(-b^2x^2-2\,abx-a^2)}}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*(2*a^2 + 1)*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b)*e^(b^2*x^2 +
 2*a*b*x + a^2) + sqrt(pi)*(2*a^2 - 1)*sqrt(b^2)*erfi(sqrt(b^2)*(b*x + a)/b
)*e^(b^2*x^2 + 2*a*b*x + a^2) - 2*b^2*x + 2*a*b + 2*(b^2*x - a*b)*e^(2*b^2*
x^2 + 4*a*b*x + 2*a^2))*e^(-b^2*x^2 - 2*a*b*x - a^2)/b^4
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cosh\left(a^2+2abx+b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh((b*x+a)**2),x)
```

```
[Out] Integral(x**2*cosh(a**2 + 2*a*b*x + b**2*x**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 137, normalized size = 1.21

$$-\frac{\frac{i\sqrt{\pi}\,(2a^2-1)\,\mathrm{erf}\left(ib\left(x+\frac{a}{b}\right)\right)}{b}-\frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(b^2x^2+2abx+a^2\right)}}{b}}{8b^2}-\frac{\frac{\sqrt{\pi}\,(2a^2+1)\,\mathrm{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}+\frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(-b^2x^2-2abx-a^2\right)}}{b}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/8*(I*sqrt(pi)*(2*a^2 - 1)*erf(I*b*(x + a/b))/b - 2*(b*(x + a/b) - 2*a)*e
^(b^2*x^2 + 2*a*b*x + a^2)/b)/b^2 - 1/8*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a
/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\cosh\left((a+b\,x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh((a + b*x)^2),x)
```

```
[Out] int(x^2*cosh((a + b*x)^2), x)
```

3.55 $\int x \cosh\left((a+bx)^2\right)\,dx$

Optimal. Leaf size=54

$$-\frac{a\sqrt{\pi}\,\text{Erf}(a+bx)}{4b^2}-\frac{a\sqrt{\pi}\,\text{Erfi}(a+bx)}{4b^2}+\frac{\sinh\left((a+bx)^2\right)}{2b^2}$$

```
[Out] 1/2*sinh((b*x+a)^2)/b^2-1/4*a*erf(b*x+a)*Pi^(1/2)/b^2-1/4*a*erfi(b*x+a)*Pi^
(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.700$, Rules used = {5473, 6874, 5407, 2235, 2236, 5429, 2717}

$$-\frac{\sqrt{\pi}\,a\text{Erf}(a+bx)}{4b^2}-\frac{\sqrt{\pi}\,a\text{Erfi}(a+bx)}{4b^2}+\frac{\sinh\left((a+bx)^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[(a + b*x)^2],x]
```

```
[Out] -1/4*(a*Sqrt[Pi]*Erf[a + b*x])/b^2 - (a*Sqrt[Pi]*Erfi[a + b*x])/(4*b^2) + S
inh[(a + b*x)^2]/(2*b^2)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x\cosh\left((a+bx)^2\right)\,dx &= \frac{\text{Subst}\left(\int(-a+x)\cosh\left(x^2\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int\left(-a\cosh\left(x^2\right)+x\cosh\left(x^2\right)\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int x\cosh\left(x^2\right)\,dx,x,a+bx\right)}{b^2}-\frac{a\text{Subst}\left(\int\cosh\left(x^2\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int\cosh(x)\,dx,x,(a+bx)^2\right)}{2b^2}-\frac{a\text{Subst}\left(\int e^{-x^2}\,dx,x,a+bx\right)}{2b^2}-\frac{a\text{Subst}\left(\int\right.}{}\\
&= -\frac{a\sqrt{\pi}\,\text{erf}(a+bx)}{4b^2}-\frac{a\sqrt{\pi}\,\text{erfi}(a+bx)}{4b^2}+\frac{\sinh\left((a+bx)^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.72

$$\frac{-a\sqrt{\pi}\left(\text{Erf}(a+bx)+\text{Erfi}(a+bx)\right)+2\sinh\left((a+bx)^2\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[(a + b*x)^2],x]
```

[Out] (-(a*Sqrt[Pi]*(Erf[a + b*x] + Erfi[a + b*x])) + 2*Sinh[(a + b*x)^2])/(4*b^2
)

Maple [C] Result contains complex when optimal does not.
time = 0.77, size = 66, normalized size = 1.22

method	result	size
risch	$-\frac{\mathrm{e}^{-(bx+a)^2}}{4b^2}-\frac{a\,\mathrm{erf}(bx+a)\sqrt{\pi}}{4b^2}+\frac{\mathrm{e}^{(bx+a)^2}}{4b^2}+\frac{ia\sqrt{\pi}\,\mathrm{erf}(ibx+ia)}{4b^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cosh((b*x+a)^2),x,method=_RETURNVERBOSE)

[Out] -1/4/b^2*exp(-(b*x+a)^2)-1/4*a*erf(b*x+a)*Pi^(1/2)/b^2+1/4/b^2*exp((b*x+a)^
2)+1/4*I*a/b^2*Pi^(1/2)*erf(I*b*x+I*a)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 650 vs.
2(44) = 88.
time = 0.46, size = 650, normalized size = 12.04

$$\frac{1}{2}\,x^2\cosh\left((bx+a)^2\right)-\frac{1}{4}\left(\frac{\left(\frac{\sqrt{\pi}\,(b^2x+ab)a^2b^3\left(\mathrm{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}\,(-b^2)^{\frac{5}{2}}}-\frac{(b^2x+ab)^3b^3\Gamma\left(\frac{3}{2},\frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{3}{2}}(-b^2)^{\frac{5}{2}}}+\frac{2ab^3e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{5}{2}}}\right)a}{\sqrt{-b^2}}+\frac{\left(\frac{\sqrt{\pi}\,(b^2x+ab)a^3b^4\left(\mathrm{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}\,(-b^2)^{\frac{7}{2}}}-\frac{3\,(b^2x+ab)^3ab^4\Gamma\left(\frac{3}{2},\frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{3}{2}}(-b^2)^{\frac{7}{2}}}+\frac{3\,a^2b^4e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{7}{2}}}+\frac{b^4\Gamma\left(2,\frac{(b^2x+ab)^2}{b^2}\right)}{(-b^2)^{\frac{7}{2}}}\right)b}{\sqrt{-b^2}}+\frac{a\left(\frac{\sqrt{\pi}\,(b^2x+ab)a^2\left(\mathrm{erf}\left(\sqrt{-\frac{(b^2x+ab)^2}{b^2}}\right)-1\right)}{b^3\sqrt{-\frac{(b^2x+ab)^2}{b^2}}}-\frac{2\,ae^{\left(\frac{(b^2x+ab)^2}{b^2}\right)}}{b^2}-\frac{(b^2x+ab)^3\Gamma\left(\frac{3}{2},-\frac{(b^2x+ab)^2}{b^2}\right)}{b^5\left(-\frac{(b^2x+ab)^2}{b^2}\right)^{\frac{3}{2}}}\right)}{b}-\left(\frac{\sqrt{\pi}\,(b^2x+ab)a^3\left(\mathrm{erf}\left(\sqrt{-\frac{(b^2x+ab)^2}{b^2}}\right)-1\right)}{b^4\sqrt{-\frac{(b^2x+ab)^2}{b^2}}}+\frac{3\,a^2e^{\left(\frac{(b^2x+ab)^2}{b^2}\right)}}{b^3}-\frac{\Gamma\left(2,-\frac{(b^2x+ab)^2}{b^2}\right)}{b^3}+\frac{3\,(b^2x+ab)^3a\Gamma\left(\frac{3}{2},-\frac{(b^2x+ab)^2}{b^2}\right)}{b^6\left(-\frac{(b^2x+ab)^2}{b^2}\right)^{\frac{3}{2}}}\right)b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cosh((b*x+a)^2),x, algorithm="maxima")

[Out] 1/2*x^2*cosh((b*x + a)^2) - 1/4*((sqrt(pi)*(b^2*x + a*b)*a^2*b^3*(erf(sqrt(
(b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(5/2)) - (b^2*x + a*
b)^3*b^3*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(5
/2)) + 2*a*b^3*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(5/2))*a/sqrt(-b^2) + (sqrt(
pi)*(b^2*x + a*b)*a^3*b^4*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x +
 a*b)^2)*(-b^2)^(7/2)) - 3*(b^2*x + a*b)^3*a*b^4*gamma(3/2, (b^2*x + a*b)^2
/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(7/2)) + 3*a^2*b^4*e^(-(b^2*x + a*b)^
2/b^2)/(-b^2)^(7/2) + b^4*gamma(2, (b^2*x + a*b)^2/b^2)/(-b^2)^(7/2))*b/sqr
t(-b^2) + a*(sqrt(pi)*(b^2*x + a*b)*a^2*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) -
1)/(b^3*sqrt(-(b^2*x + a*b)^2/b^2)) - 2*a*e^((b^2*x + a*b)^2/b^2)/b^2 - (b^
2*x + a*b)^3*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^5*(-(b^2*x + a*b)^2/b^2)^(
3/2)))/b - sqrt(pi)*(b^2*x + a*b)*a^3*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)
/(b^4*sqrt(-(b^2*x + a*b)^2/b^2)) + 3*a^2*e^((b^2*x + a*b)^2/b^2)/b^3 - gam
ma(2, -(b^2*x + a*b)^2/b^2)/b^3 + 3*(b^2*x + a*b)^3*a*gamma(3/2, -(b^2*x +
a*b)^2/b^2)/(b^6*(-(b^2*x + a*b)^2/b^2)^(3/2)))*b

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs.
2(44) = 88.
time = 0.44, size = 134, normalized size = 2.48

$$-\frac{\left(\sqrt{\pi}\,a\sqrt{b^2}\,\mathrm{erf}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)}+\sqrt{\pi}\,a\sqrt{b^2}\,\mathrm{erfi}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)}-be^{\left(2\,b^2x^2+4\,abx+2\,a^2\right)}+b\right)e^{\left(-b^2x^2-2\,abx-a^2\right)}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*a*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b)*e^(b^2*x^2 + 2*a*b*x
+ a^2) + sqrt(pi)*a*sqrt(b^2)*erfi(sqrt(b^2)*(b*x + a)/b)*e^(b^2*x^2 + 2*a*
b*x + a^2) - b*e^(2*b^2*x^2 + 4*a*b*x + 2*a^2) + b)*e^(-b^2*x^2 - 2*a*b*x -
 a^2)/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(a^2+2abx+b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh((b*x+a)**2),x)
```

```
[Out] Integral(x*cosh(a**2 + 2*a*b*x + b**2*x**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 99, normalized size = 1.83

$$-\frac{-\frac{i\sqrt{\pi}\,a\operatorname{erf}\left(ib\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(b^2x^2+2\,abx+a^2\right)}}{b}}{4\,b}+\frac{\frac{\sqrt{\pi}\,a\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}-\frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(pi)*a*erf(I*b*(x + a/b))/b - e^(b^2*x^2 + 2*a*b*x + a^2)/b)/b
 + 1/4*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\cosh\left({\left(a+b\,x\right)}^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh((a + b*x)^2),x)
```

```
[Out] int(x*cosh((a + b*x)^2), x)
```

3.56 $\int \cosh\left((a+bx)^2\right) dx$

Optimal. Leaf size=37

$$\frac{\sqrt{\pi}\,\mathrm{Erf}(a+bx)}{4b}+\frac{\sqrt{\pi}\,\mathrm{Erfi}(a+bx)}{4b}$$

```
[Out] 1/4*erf(b*x+a)*Pi^(1/2)/b+1/4*erfi(b*x+a)*Pi^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5419, 5407, 2235, 2236}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}(a+bx)}{4b}+\frac{\sqrt{\pi}\,\mathrm{Erfi}(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(a + b*x)^2],x]
```

```
[Out] (Sqrt[Pi]*Erf[a + b*x])/(4*b) + (Sqrt[Pi]*Erfi[a + b*x])/(4*b)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5419

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Cosh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\int \cosh \left((a+bx)^2\right) \, dx = \frac{\text{Subst}\left(\int \cosh \left(x^2\right) \, dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int e^{-x^2} \, dx, x, a+bx\right)}{2b} + \frac{\text{Subst}\left(\int e^{x^2} \, dx, x, a+bx\right)}{2b}$$
$$= \frac{\sqrt{\pi}\operatorname{erf}(a+bx)}{4b} + \frac{\sqrt{\pi}\operatorname{erfi}(a+bx)}{4b}$$

__Mathematica__ [A]
time = 0.01, size = 25, normalized size = 0.68

$$\frac{\sqrt{\pi}\,(\text{Erf}(a+bx)+\text{Erfi}(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(a + b*x)^2],x]
```

```
[Out] (Sqrt[Pi]*(Erf[a + b*x] + Erfi[a + b*x]))/(4*b)
```

__Maple__ [C] Result contains complex when optimal does not.
time = 0.78, size = 36, normalized size = 0.97

method	result	size
risch	$\frac{\operatorname{erf}(bx+a)\sqrt{\pi}}{4b} - \frac{i\sqrt{\pi}\,\operatorname{erf}(ibx+ia)}{4b}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*erf(b*x+a)*Pi^(1/2)/b-1/4*I*Pi^(1/2)/b*erf(I*b*x+I*a)
```

__Maxima__ [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(29) = 58$.
time = 0.42, size = 478, normalized size = 12.92

$$-\frac{1}{2}\left(\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)ab^2\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}\,(-b^2)^{\frac{3}{2}}} + \frac{b^2 e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{3}{2}}}\right)a}{\sqrt{-b^2}} + \frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^2b^3\left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right)-1\right)}{\sqrt{(b^2x+ab)^2}\,(-b^2)^{\frac{5}{2}}} - \frac{(b^2x+ab)^3b^3\Gamma\left(\frac{3}{2},\frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{3}{2}}(-b^2)^{\frac{5}{2}}} + \frac{2ab^3e^{\left(-\frac{(b^2x+ab)^2}{b^2}\right)}}{(-b^2)^{\frac{5}{2}}}\right)b}{\sqrt{-b^2}} - \frac{a\left(\frac{\sqrt{\pi}(b^2x+ab)a\left(\operatorname{erf}\left(\sqrt{-\frac{(b^2x+ab)^2}{b^2}}\right)-1\right)}{b^2\sqrt{-\frac{(b^2x+ab)^2}{b^2}}} - \frac{e^{\left(\frac{(b^2x+ab)^2}{b^2}\right)}}{b}\right)}{b} + \frac{\sqrt{\pi}(b^2x+ab)a^2\left(\operatorname{erf}\left(\sqrt{-\frac{(b^2x+ab)^2}{b^2}}\right)-1\right)}{b^3\sqrt{-\frac{(b^2x+ab)^2}{b^2}}} - \frac{2ae^{\left(\frac{(b^2x+ab)^2}{b^2}\right)}}{b^2} - \frac{(b^2x+ab)^3\Gamma\left(\frac{3}{2},-\frac{(b^2x+ab)^2}{b^2}\right)}{b^5\left(-\frac{(b^2x+ab)^2}{b^2}\right)^{\frac{3}{2}}}\right)b + x\cosh\left((bx+a)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/2*((sqrt(pi)*(b^2*x + a*b)*a*b^2*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqr
t((b^2*x + a*b)^2)*(-b^2)^(3/2)) + b^2*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(3/2
))*a/sqrt(-b^2) + (sqrt(pi)*(b^2*x + a*b)*a^2*b^3*(erf(sqrt((b^2*x + a*b)^2
)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(5/2)) - (b^2*x + a*b)^3*b^3*gamma(
3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(5/2)) + 2*a*b^3*
e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(5/2))*b/sqrt(-b^2) - a*(sqrt(pi)*(b^2*x +
a*b)*a*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/(b^2*sqrt(-(b^2*x + a*b)^2/b^2
)) - e^((b^2*x + a*b)^2/b^2)/b)/b + sqrt(pi)*(b^2*x + a*b)*a^2*(erf(sqrt(-(
b^2*x + a*b)^2/b^2)) - 1)/(b^3*sqrt(-(b^2*x + a*b)^2/b^2)) - 2*a*e^((b^2*x
+ a*b)^2/b^2)/b^2 - (b^2*x + a*b)^3*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^5*(
-(b^2*x + a*b)^2/b^2)^(3/2)))*b + x*cosh((b*x + a)^2)
```

Fricas [A]
time = 0.37, size = 54, normalized size = 1.46

$$\frac{\sqrt{\pi}\,\sqrt{b^2}\,\operatorname{erf}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)+\sqrt{\pi}\,\sqrt{b^2}\,\operatorname{erfi}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b) + sqrt(pi)*sqrt(b^2)*erf
i(sqrt(b^2)*(b*x + a)/b))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left((a+bx)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)**2),x)
```

```
[Out] Integral(cosh((a + b*x)**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 39, normalized size = 1.05

$$-\frac{i\,\sqrt{\pi}\,\operatorname{erf}\left(i\,b\left(x+\frac{a}{b}\right)\right)}{4\,b}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(pi)*erf(I*b*(x + a/b))/b - 1/4*sqrt(pi)*erf(-b*(x + a/b))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cosh\left((a+b\,x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((a + b*x)^2),x)
```

```
[Out] int(cosh((a + b*x)^2), x)
```

3.57 $\int \frac{\cosh\left((a+bx)^2\right)}{x}\,dx$

Optimal. Leaf size=20

$$b\mathrm{Int}\left(\frac{\cosh\left((a+bx)^2\right)}{bx},x\right)$$

```
[Out] b*CannotIntegrate(cosh((b*x+a)^2)/b/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh\left((a+bx)^2\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cosh[(a + b*x)^2]/x,x]
```

```
[Out] Defer[Subst][Defer[Int][Cosh[x^2]/(-a + x), x], x, a + b*x]
```

Rubi steps

$$\int \frac{\cosh\left((a+bx)^2\right)}{x}\,dx = \mathrm{Subst}\left(\int \frac{\cosh\left(x^2\right)}{-a+x}\,dx,x,a+bx\right)$$

Mathematica [A]
time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left((a+bx)^2\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[(a + b*x)^2]/x,x]
```

```
[Out] Integrate[Cosh[(a + b*x)^2]/x, x]
```

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(\left(bx+a\right)^{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((b*x+a)^2)/x,x)
```

```
[Out] int(cosh((b*x+a)^2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh((b*x + a)^2)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b^2*x^2 + 2*a*b*x + a^2)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a^{2} + 2abx + b^{2}x^{2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)**2)/x,x)
```

```
[Out] Integral(cosh(a**2 + 2*a*b*x + b**2*x**2)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh((b*x + a)^2)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cosh\left({\left(a+b\,x\right)}^2\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((a + b*x)^2)/x,x)
```

```
[Out] int(cosh((a + b*x)^2)/x, x)
```

3.58 $\int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cosh\left((a+bx)^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable(cosh((b*x+a)^2)/x^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cosh[(a + b*x)^2]/x^2,x]
[Out] b*Defer[Subst][Defer[Int][Cosh[x^2]/(-a + x)^2, x], x, a + b*x]
```

Rubi steps

$$\int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx = b\text{Subst}\left(\int \frac{\cosh\left(x^2\right)}{(-a+x)^2}\,dx,x,a+bx\right)$$

Mathematica [A]
time = 7.43, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left((a+bx)^2\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[(a + b*x)^2]/x^2,x]
[Out] Integrate[Cosh[(a + b*x)^2]/x^2, x]
```

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((b*x+a)^2)/x^2,x)
[Out] int(cosh((b*x+a)^2)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)^2)/x^2,x, algorithm="maxima")
[Out] integrate(cosh((b*x + a)^2)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)^2)/x^2,x, algorithm="fricas")
[Out] integral(cosh(b^2*x^2 + 2*a*b*x + a^2)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh \left(a^2 + 2abx + b^2x^2\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)**2)/x**2,x)
[Out] Integral(cosh(a**2 + 2*a*b*x + b**2*x**2)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)^2)/x^2,x, algorithm="giac")
[Out] integrate(cosh((b*x + a)^2)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cosh\left({\left(a+b\,x\right)}^2\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((a + b*x)^2)/x^2,x)
```

```
[Out] int(cosh((a + b*x)^2)/x^2, x)
```

3.59 $\int x^2 \cosh\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=346

$$-\frac{240\cosh\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{24c\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}-\frac{2c^2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120(c+dx)\cosh\left(a-\right.}{b^4d^3}$$

```
[Out] -240*cosh(a+b*(d*x+c)^(1/2))/b^6/d^3+24*c*cosh(a+b*(d*x+c)^(1/2))/b^4/d^3-2
*c^2*cosh(a+b*(d*x+c)^(1/2))/b^2/d^3-120*(d*x+c)*cosh(a+b*(d*x+c)^(1/2))/b^
4/d^3+12*c*(d*x+c)*cosh(a+b*(d*x+c)^(1/2))/b^2/d^3-10*(d*x+c)^2*cosh(a+b*(d
*x+c)^(1/2))/b^2/d^3+40*(d*x+c)^(3/2)*sinh(a+b*(d*x+c)^(1/2))/b^3/d^3-4*c*(
d*x+c)^(3/2)*sinh(a+b*(d*x+c)^(1/2))/b/d^3+2*(d*x+c)^(5/2)*sinh(a+b*(d*x+c)
^(1/2))/b/d^3+240*sinh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^5/d^3-24*c*sinh(a
+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^3+2*c^2*sinh(a+b*(d*x+c)^(1/2))*(d*x+
c)^(1/2)/b/d^3
```

Rubi [A]
time = 0.30, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5473, 5395, 3377, 2718}

$$-\frac{240\cosh\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{240\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{b^5d^3}-\frac{120(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{24c\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{40(c+dx)^{3/2}\sinh\left(a+b\sqrt{c+dx}\right)}{b^3d^3}-\frac{24c\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{b^3d^3}-\frac{2c^2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{10(c+dx)^2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{12c(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{2c^2\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{bd^3}+\frac{2(c+dx)^{5/2}\sinh\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{4c(c+dx)^{3/2}\sinh\left(a+b\sqrt{c+dx}\right)}{bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-240*Cosh[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (24*c*Cosh[a + b*Sqrt[c + d*x]
])/(b^4*d^3) - (2*c^2*Cosh[a + b*Sqrt[c + d*x]])/(b^2*d^3) - (120*(c + d*x)
*Cosh[a + b*Sqrt[c + d*x]])/(b^4*d^3) + (12*c*(c + d*x)*Cosh[a + b*Sqrt[c +
 d*x]])/(b^2*d^3) - (10*(c + d*x)^2*Cosh[a + b*Sqrt[c + d*x]])/(b^2*d^3) +
(240*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c + d*x]])/(b^5*d^3) - (24*c*Sqrt[c + d*
x]*Sinh[a + b*Sqrt[c + d*x]])/(b^3*d^3) + (2*c^2*Sqrt[c + d*x]*Sinh[a + b*S
qrt[c + d*x]])/(b*d^3) + (40*(c + d*x)^(3/2)*Sinh[a + b*Sqrt[c + d*x]])/(b^
3*d^3) - (4*c*(c + d*x)^(3/2)*Sinh[a + b*Sqrt[c + d*x]])/(b*d^3) + (2*(c +
d*x)^(5/2)*Sinh[a + b*Sqrt[c + d*x]])/(b*d^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cosh\left(a+b\sqrt{c+dx}\right)\,dx &= \frac{\text{Subst}\left(\int (-c+x)^2 \cosh\left(a+b\sqrt{x}\right)\,dx, x, c+dx\right)}{d^3}\\
&= \frac{2\text{Subst}\left(\int x\left(c-x^2\right)^2 \cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3}\\
&= \frac{2\text{Subst}\left(\int \left(c^2 x\cosh(a+bx)-2cx^3\cosh(a+bx)+x^5\cosh(a+bx)\right)\,dx, x\right.}{d^3}\\
&= \frac{2\text{Subst}\left(\int x^5\cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3}-\frac{(4c)\text{Subst}\left(\int x^3\cosh(a+b\right.}{d^3}\\
&= \frac{2c^2\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{4c(c+dx)^{3/2}\sinh\left(a+b\sqrt{c+dx}\right.}{bd^3}\\
&= -\frac{2c^2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{12c(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{10(}{}\\
&= -\frac{2c^2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{12c(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{10(}{}\\
&= \frac{24c\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}-\frac{2c^2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120(c+dx)\,\text{co}}{}\\
&= \frac{24c\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}-\frac{2c^2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120(c+dx)\,\text{co}}{}\\
&= -\frac{240\cosh\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{24c\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^3}-\frac{2c^2\cosh\left(a\right.}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.38, size = 104, normalized size = 0.30

$$\frac{-2(120+12b^2(4c+5dx)+b^4dx(4c+5dx))\cosh\left(a+b\sqrt{c+dx}\right)+2b\sqrt{c+dx}\,(120+b^4d^2x^2+4b^2(2c+5dx))\sinh\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*(120 + 12*b^2*(4*c + 5*d*x) + b^4*d*x*(4*c + 5*d*x))*Cosh[a + b*Sqrt[c
+ d*x]] + 2*b*Sqrt[c + d*x]*(120 + b^4*d^2*x^2 + 4*b^2*(2*c + 5*d*x))*Sinh[
a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. 2(310) = 620.
time = 1.66, size = 831, normalized size = 2.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3/b^2*(5/b^4*a^4*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*
(d*x+c)^(1/2)))-1/b^4*a^5*sinh(a+b*(d*x+c)^(1/2))-10/b^4*a^3*((a+b*(d*x+c)^
(1/2))^2*sinh(a+b*(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/
2))+2*sinh(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*((a+b*(d*x+c)^(1/2))^3*sinh(a+b*(
d*x+c)^(1/2))-3*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c
)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-6*cosh(a+b*(d*x+c)^(1/2)))-6/b^2*a^2*c*((a
+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2)))+2/b^2*a^
3*c*sinh(a+b*(d*x+c)^(1/2))-5/b^4*a*((a+b*(d*x+c)^(1/2))^4*sinh(a+b*(d*x+c)
^(1/2))-4*(a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))+12*(a+b*(d*x+c)^(1/
2))^2*sinh(a+b*(d*x+c)^(1/2))-24*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2)
)+24*sinh(a+b*(d*x+c)^(1/2)))+6/b^2*a*c*((a+b*(d*x+c)^(1/2))^2*sinh(a+b*(d*
x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))+2*sinh(a+b*(d*x+c
)^(1/2)))+1/b^4*((a+b*(d*x+c)^(1/2))^5*sinh(a+b*(d*x+c)^(1/2))-5*(a+b*(d*x+
c)^(1/2))^4*cosh(a+b*(d*x+c)^(1/2))+20*(a+b*(d*x+c)^(1/2))^3*sinh(a+b*(d*x+
c)^(1/2))-60*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))+120*(a+b*(d*x+c)
^(1/2))*sinh(a+b*(d*x+c)^(1/2))-120*cosh(a+b*(d*x+c)^(1/2)))-2/b^2*c*((a+b*
(d*x+c)^(1/2))^3*sinh(a+b*(d*x+c)^(1/2))-3*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(
d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-6*cosh(a+b*(d*x
+c)^(1/2)))+c^2*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+
c)^(1/2)))-c^2*a*sinh(a+b*(d*x+c)^(1/2)))
```

Maxima [A]
time = 0.27, size = 486, normalized size = 1.40

$$\frac{2\,d^3x^3\cosh\left(\sqrt{dx+c}\,b+a\right)+\left(\frac{c^3e^{\left(\sqrt{dx+c}\,b+a\right)}}{b}+\frac{c^3e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b}-\frac{3\left((dx+c)b^2e^a-2\sqrt{dx+c}\,be^a+2e^a\right)c^2e^{\left(\sqrt{dx+c}\,b\right)}}{b^3}-\frac{3\left((dx+c)b^2+2\sqrt{dx+c}\,b+2\right)c^2e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^3}+\frac{3\left((dx+c)^2b^4e^a-4(dx+c)^{\frac{3}{2}}b^3e^a+12(dx+c)b^2e^a-24\sqrt{dx+c}\,be^a+24e^a\right)ce^{\left(\sqrt{dx+c}\,b\right)}}{b^5}+\frac{3\left((dx+c)^2b^4+4(dx+c)^{\frac{3}{2}}b^3+12(dx+c)b^2+24\sqrt{dx+c}\,b+24\right)ce^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^5}-\frac{\left((dx+c)^3b^6e^a-6(dx+c)^{\frac{5}{2}}b^5e^a+30(dx+c)^2b^4e^a-120(dx+c)^{\frac{3}{2}}b^3e^a+360(dx+c)b^2e^a-720\sqrt{dx+c}\,be^a+720e^a\right)e^{\left(\sqrt{dx+c}\,b\right)}}{b^7}-\frac{\left((dx+c)^3b^6+6(dx+c)^{\frac{5}{2}}b^5+30(dx+c)^2b^4+120(dx+c)^{\frac{3}{2}}b^3+360(dx+c)b^2+720\sqrt{dx+c}\,b+720\right)e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^7}\right)b}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*d^3*x^3*cosh(sqrt(d*x + c)*b + a) + (c^3*e^(sqrt(d*x + c)*b + a)/b +
 c^3*e^(-sqrt(d*x + c)*b - a)/b - 3*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*
e^a + 2*e^a)*c^2*e^(sqrt(d*x + c)*b)/b^3 - 3*((d*x + c)*b^2 + 2*sqrt(d*x +
c)*b + 2)*c^2*e^(-sqrt(d*x + c)*b - a)/b^3 + 3*((d*x + c)^2*b^4*e^a - 4*(d*
x + c)^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e
^a)*c*e^(sqrt(d*x + c)*b)/b^5 + 3*((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3
+ 12*(d*x + c)*b^2 + 24*sqrt(d*x + c)*b + 24)*c*e^(-sqrt(d*x + c)*b - a)/b^
5 - ((d*x + c)^3*b^6*e^a - 6*(d*x + c)^(5/2)*b^5*e^a + 30*(d*x + c)^2*b^4*e
^a - 120*(d*x + c)^(3/2)*b^3*e^a + 360*(d*x + c)*b^2*e^a - 720*sqrt(d*x + c
)*b*e^a + 720*e^a)*e^(sqrt(d*x + c)*b)/b^7 - ((d*x + c)^3*b^6 + 6*(d*x + c)
^(5/2)*b^5 + 30*(d*x + c)^2*b^4 + 120*(d*x + c)^(3/2)*b^3 + 360*(d*x + c)*b
^2 + 720*sqrt(d*x + c)*b + 720)*e^(-sqrt(d*x + c)*b - a)/b^7)*b)/d^3
```

Fricas [A]
time = 0.50, size = 104, normalized size = 0.30

$$\frac{2\left(\left(b^{5}d^{2}x^{2}+20\,b^{3}dx+8\,b^{3}c+120\,b\right)\sqrt{dx+c}\,\sinh\left(\sqrt{dx+c}\,b+a\right)-\left(5\,b^{4}d^{2}x^{2}+48\,b^{2}c+4\,(b^{4}c+15\,b^{2})dx+120\right)\cosh\left(\sqrt{dx+c}\,b+a\right)\right)}{b^{6}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^5*d^2*x^2 + 20*b^3*d*x + 8*b^3*c + 120*b)*sqrt(d*x + c)*sinh(sqrt(d*x
 + c)*b + a) - (5*b^4*d^2*x^2 + 48*b^2*c + 4*(b^4*c + 15*b^2)*d*x + 120)*co
sh(sqrt(d*x + c)*b + a))/(b^6*d^3)
```

Sympy [A]
time = 0.32, size = 269, normalized size = 0.78

$$\begin{cases} \frac{x^{3}\cosh{\left(a \right)}}{3} & \text{for}\: b = 0 \wedge \left(b = 0 \vee d = 0\right) \\ \frac{x^{3}\cosh{\left(a + b\sqrt{c} \right)}}{3} & \text{for}\: d = 0 \\ \frac{2x^{2}\sqrt{c+dx}\,\sinh{\left(a + b\sqrt{c+dx} \right)}}{bd} - \frac{8cx\cosh{\left(a + b\sqrt{c+dx} \right)}}{b^{2}d^{2}} - \frac{10x^{2}\cosh{\left(a + b\sqrt{c+dx} \right)}}{b^{2}d} + \frac{16c\sqrt{c+dx}\,\sinh{\left(a + b\sqrt{c+dx} \right)}}{b^{3}d^{3}} + \frac{40x\sqrt{c+dx}\,\sinh{\left(a + b\sqrt{c+dx} \right)}}{b^{3}d^{2}} - \frac{96c\cosh{\left(a + b\sqrt{c+dx} \right)}}{b^{4}d^{3}} - \frac{120x\cosh{\left(a + b\sqrt{c+dx} \right)}}{b^{4}d^{2}} + \frac{240\sqrt{c+dx}\,\sinh{\left(a + b\sqrt{c+dx} \right)}}{b^{5}d^{3}} - \frac{240\cosh{\left(a + b\sqrt{c+dx} \right)}}{b^{6}d^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**3*cosh(a)/3, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**3*cosh(a
+ b*sqrt(c))/3, Eq(d, 0)), (2*x**2*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/
(b*d) - 8*c*x*cosh(a + b*sqrt(c + d*x))/(b**2*d**2) - 10*x**2*cosh(a + b*sq
rt(c + d*x))/(b**2*d) + 16*c*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**3*
d**3) + 40*x*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*cos
h(a + b*sqrt(c + d*x))/(b**4*d**3) - 120*x*cosh(a + b*sqrt(c + d*x))/(b**4*
d**2) + 240*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**5*d**3) - 240*cosh(
a + b*sqrt(c + d*x))/(b**6*d**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(310) = 620.

time = 0.42, size = 915, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] (((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b + a)^3*b^2
*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^2*b^2*c
+ 2*a^3*b^2*c - b^4*c^2 + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)
^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*
(sqrt(d*x + c)*b + a)*a^4 - a^5 + 6*(sqrt(d*x + c)*b + a)^2*b^2*c - 12*(sqr
t(d*x + c)*b + a)*a*b^2*c + 6*a^2*b^2*c - 5*(sqrt(d*x + c)*b + a)^4 + 20*(s
qrt(d*x + c)*b + a)^3*a - 30*(sqrt(d*x + c)*b + a)^2*a^2 + 20*(sqrt(d*x + c
)*b + a)*a^3 - 5*a^4 - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*c + 20*(sq
rt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*x + c)*b +
 a)*a^2 - 20*a^3 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(sqrt(d*x +
c)*b + a)*a - 60*a^2 + 120*sqrt(d*x + c)*b - 120)*e^(sqrt(d*x + c)*b + a)/(
b^5*d^2) - ((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b
+ a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*
a^2*b^2*c + 2*a^3*b^2*c + b^4*c^2 + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x +
 c)*b + a)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^
2*a^3 + 5*(sqrt(d*x + c)*b + a)*a^4 - a^5 - 6*(sqrt(d*x + c)*b + a)^2*b^2*c
 + 12*(sqrt(d*x + c)*b + a)*a*b^2*c - 6*a^2*b^2*c + 5*(sqrt(d*x + c)*b + a)
^4 - 20*(sqrt(d*x + c)*b + a)^3*a + 30*(sqrt(d*x + c)*b + a)^2*a^2 - 20*(sq
rt(d*x + c)*b + a)*a^3 + 5*a^4 - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*
c + 20*(sqrt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*
x + c)*b + a)*a^2 - 20*a^3 - 12*b^2*c + 60*(sqrt(d*x + c)*b + a)^2 - 120*(s
qrt(d*x + c)*b + a)*a + 60*a^2 + 120*sqrt(d*x + c)*b + 120)*e^(-sqrt(d*x +
c)*b - a)/(b^5*d^2))/(b*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\cosh\left(a+b\sqrt{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] int(x^2*cosh(a + b*(c + d*x)^(1/2)), x)
```

3.60 $\int x\cosh\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=167

$$-\frac{12\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2c\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{6(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{12\sqrt{c+dx}\,\sinh}{b}$$

```
[Out] -12*cosh(a+b*(d*x+c)^(1/2))/b^4/d^2+2*c*cosh(a+b*(d*x+c)^(1/2))/b^2/d^2-6*(
d*x+c)*cosh(a+b*(d*x+c)^(1/2))/b^2/d^2+2*(d*x+c)^(3/2)*sinh(a+b*(d*x+c)^(1/
2))/b/d^2+12*sinh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^2-2*c*sinh(a+b*(d*
x+c)^(1/2))*(d*x+c)^(1/2)/b/d^2
```

Rubi [A]
time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5473, 5395, 3377, 2718}

$$-\frac{12\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{12\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{6(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{2c\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{2(c+dx)^{3/2}\sinh\left(a+b\sqrt{c+dx}\right)}{bd^2}-\frac{2c\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-12*Cosh[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (2*c*Cosh[a + b*Sqrt[c + d*x]])
/(b^2*d^2) - (6*(c + d*x)*Cosh[a + b*Sqrt[c + d*x]])/(b^2*d^2) + (12*Sqrt[c
 + d*x]*Sinh[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*c*Sqrt[c + d*x]*Sinh[a +
b*Sqrt[c + d*x]])/(b*d^2) + (2*(c + d*x)^(3/2)*Sinh[a + b*Sqrt[c + d*x]])/(
b*d^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\int x\cosh\left(a+b\sqrt{c+dx}\right)\,dx=\frac{\text{Subst}\left(\int(-c+x)\cosh\left(a+b\sqrt{x}\right)\,dx,x,c+dx\right)}{d^2}$$
$$=\frac{2\text{Subst}\left(\int x(-c+x^2)\cosh(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$=\frac{2\text{Subst}\left(\int\left(-cx\cosh(a+bx)+x^3\cosh(a+bx)\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$=\frac{2\text{Subst}\left(\int x^3\cosh(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2}-\frac{(2c)\text{Subst}\left(\int x\cosh(a+bx)\right.}{d^2}$$
$$=-\frac{2c\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2(c+dx)^{3/2}\sinh\left(a+b\sqrt{c+dx}\right)}{bd^2}$$
$$=\frac{2c\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{6(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{2c\sqrt{c+dx}}{}$$
$$=\frac{2c\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{6(c+dx)\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{12\sqrt{c+dx}}{}$$
$$=-\frac{12\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2c\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{6(c+dx)\cosh\left(\right.}{b^2}$$

Mathematica [A]
time = 0.14, size = 72, normalized size = 0.43

$$\frac{-2(6+b^2(2c+3dx))\cosh\left(a+b\sqrt{c+dx}\right)+2b\sqrt{c+dx}\,(6+b^2dx)\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*(6 + b^2*(2*c + 3*d*x))*Cosh[a + b*Sqrt[c + d*x]] + 2*b*Sqrt[c + d*x]*(
6 + b^2*d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(149) = 298.
time = 1.65, size = 303, normalized size = 1.81

method	result
derivativedivides	$\frac{\frac{6a^2\left(\left(a+b\sqrt{dx+c}\right)\sinh\left(a+b\sqrt{dx+c}\right)-\cosh\left(a+b\sqrt{dx+c}\right)\right)}{b^2}-\frac{2a^3\sinh\left(a+b\sqrt{dx+c}\right)}{b^2}-\frac{6a\left(\left(a+b\sqrt{c\right.}\right.}{}}{}$
default	$\frac{\frac{6a^2\left(\left(a+b\sqrt{dx+c}\right)\sinh\left(a+b\sqrt{dx+c}\right)-\cosh\left(a+b\sqrt{dx+c}\right)\right)}{b^2}-\frac{2a^3\sinh\left(a+b\sqrt{dx+c}\right)}{b^2}-\frac{6a\left(\left(a+b\sqrt{c\right.}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2/b^2*(3/b^2*a^2*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*
(d*x+c)^(1/2)))-1/b^2*a^3*sinh(a+b*(d*x+c)^(1/2))-3/b^2*a*((a+b*(d*x+c)^(1/
2))^2*sinh(a+b*(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))
+2*sinh(a+b*(d*x+c)^(1/2)))+1/b^2*((a+b*(d*x+c)^(1/2))^3*sinh(a+b*(d*x+c)^(
1/2))-3*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))
*sinh(a+b*(d*x+c)^(1/2))-6*cosh(a+b*(d*x+c)^(1/2)))-c*((a+b*(d*x+c)^(1/2))*
sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2)))+c*a*sinh(a+b*(d*x+c)^(1/2)
))
```

Maxima [A]
time = 0.28, size = 291, normalized size = 1.74

$$\frac{2\,d^2x^2\cosh\left(\sqrt{dx+c}\,b+a\right)-\left(\frac{c^2e^{\left(\sqrt{dx+c}\,b+a\right)}}{b}+\frac{c^2e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b}-\frac{2\left((dx+c)b^2e^a-2\sqrt{dx+c}\,be^a+2\,e^a\right)ce^{\left(\sqrt{dx+c}\,b\right)}}{b^3}-\frac{2\left((dx+c)b^2+2\sqrt{dx+c}\,b+2\right)ce^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^3}+\frac{\left((dx+c)^2b^4e^a-4\,(dx+c)^{\frac{3}{2}}b^3e^a+12\,(dx+c)b^2e^a-24\sqrt{dx+c}\,be^a+24\,e^a\right)e^{\left(\sqrt{dx+c}\,b\right)}}{b^5}+\frac{\left((dx+c)^2b^4+4\,(dx+c)^{\frac{3}{2}}b^3+12\,(dx+c)b^2+24\sqrt{dx+c}\,b+24\right)e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^5}\right)b}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d^2*x^2*cosh(sqrt(d*x + c)*b + a) - (c^2*e^(sqrt(d*x + c)*b + a)/b +
 c^2*e^(-sqrt(d*x + c)*b - a)/b - 2*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*
e^a + 2*e^a)*c*e^(sqrt(d*x + c)*b)/b^3 - 2*((d*x + c)*b^2 + 2*sqrt(d*x + c)
*b + 2)*c*e^(-sqrt(d*x + c)*b - a)/b^3 + ((d*x + c)^2*b^4*e^a - 4*(d*x + c)
^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e^a)*e^
(sqrt(d*x + c)*b)/b^5 + ((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3 + 12*(d*x
+ c)*b^2 + 24*sqrt(d*x + c)*b + 24)*e^(-sqrt(d*x + c)*b - a)/b^5)*b)/d^2
```

Fricas [A]
time = 0.40, size = 68, normalized size = 0.41

$$\frac{2\left(\left(b^3dx+6\,b\right)\sqrt{dx+c}\,\sinh\left(\sqrt{dx+c}\,b+a\right)-\left(3\,b^2dx+2\,b^2c+6\right)\cosh\left(\sqrt{dx+c}\,b+a\right)\right)}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^3*d*x + 6*b)*sqrt(d*x + c)*sinh(sqrt(d*x + c)*b + a) - (3*b^2*d*x + 2
*b^2*c + 6)*cosh(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

Sympy [A]
time = 0.17, size = 151, normalized size = 0.90

$$\begin{cases} \frac{x^2\cosh(a)}{2} & \text{for } b=0 \wedge (b=0 \vee d=0) \\ \frac{x^2\cosh\left(a+b\sqrt{c}\right)}{2} & \text{for } d=0 \\ \frac{2x\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{4c\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{6x\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{12\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{12\cosh\left(a+b\sqrt{c+dx}\right)}{b^4d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**2*cosh(a)/2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**2*cosh(a
+ b*sqrt(c))/2, Eq(d, 0)), (2*x*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b*
d) - 4*c*cosh(a + b*sqrt(c + d*x))/(b**2*d**2) - 6*x*cosh(a + b*sqrt(c + d*
x))/(b**2*d) + 12*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*
cosh(a + b*sqrt(c + d*x))/(b**4*d**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(149) = 298.
time = 0.42, size = 300, normalized size = 1.80

$$-\frac{\frac{\left(\left(\sqrt{dx+c}\,b+a\right)b^2c-ab^2c-\left(\sqrt{dx+c}\,b+a\right)^3+3\left(\sqrt{dx+c}\,b+a\right)^2a-3\left(\sqrt{dx+c}\,b+a\right)a^2+a^3-b^2c+3\left(\sqrt{dx+c}\,b+a\right)^2-6\left(\sqrt{dx+c}\,b+a\right)a+3a^2-6\sqrt{dx+c}\,b+6\right)e^{\left(\sqrt{dx+c}\,b+a\right)}}{b^3d}-\frac{\left(\left(\sqrt{dx+c}\,b+a\right)b^2c-ab^2c-\left(\sqrt{dx+c}\,b+a\right)^3+3\left(\sqrt{dx+c}\,b+a\right)^2a-3\left(\sqrt{dx+c}\,b+a\right)a^2+a^3+b^2c-3\left(\sqrt{dx+c}\,b+a\right)^2+6\left(\sqrt{dx+c}\,b+a\right)a-3a^2-6\sqrt{dx+c}\,b-6\right)e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^3d}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -((((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqr
t(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 - b^2*c + 3*(sqrt
(d*x + c)*b + a)^2 - 6*(sqrt(d*x + c)*b + a)*a + 3*a^2 - 6*sqrt(d*x + c)*b
+ 6)*e^(sqrt(d*x + c)*b + a)/(b^3*d) - ((sqrt(d*x + c)*b + a)*b^2*c - a*b^2
*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x +
c)*b + a)*a^2 + a^3 + b^2*c - 3*(sqrt(d*x + c)*b + a)^2 + 6*(sqrt(d*x + c)*
b + a)*a - 3*a^2 - 6*sqrt(d*x + c)*b - 6)*e^(-sqrt(d*x + c)*b - a)/(b^3*d))
/(b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\cosh\left(a+b\sqrt{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] int(x*cosh(a + b*(c + d*x)^(1/2)), x)
```

3.61 $\int \cosh\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=54

$$-\frac{2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{2\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{bd}$$

```
[Out] -2*cosh(a+b*(d*x+c)^(1/2))/b^2/d+2*sinh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/
d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {5419, 5413, 3377, 2718}

$$\frac{2\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-2*Cosh[a + b*Sqrt[c + d*x]])/(b^2*d) + (2*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c
 + d*x]])/(b*d)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5413

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Cosh[c + d*x^(k*
n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && I
ntegerQ[p]
```

Rule 5419

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Cosh[c + d*x^n])^p, x], x, u], x]
```

```
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cosh\left(a+b\sqrt{c+dx}\right)\,dx &= \frac{\text{Subst}\left(\int \cosh\left(a+b\sqrt{x}\right)\,dx, x, c+dx\right)}{d}\\
&= \frac{2\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= \frac{2\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd}\\
&= -\frac{2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{2\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)}{bd}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.93

$$\frac{2\left(-\cosh\left(a+b\sqrt{c+dx}\right)+b\sqrt{c+dx}\,\sinh\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*(-Cosh[a + b*Sqrt[c + d*x]] + b*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c + d*x]])
)/(b^2*d)
```

Maple [A]
time = 1.39, size = 63, normalized size = 1.17

method	result	size
derivativedivides	$\frac{2\left(a+b\sqrt{dx+c}\right)\sinh\left(a+b\sqrt{dx+c}\right)-2\cosh\left(a+b\sqrt{dx+c}\right)-2a\sinh\left(a+b\sqrt{dx+c}\right)}{b^2d}$	63
default	$\frac{2\left(a+b\sqrt{dx+c}\right)\sinh\left(a+b\sqrt{dx+c}\right)-2\cosh\left(a+b\sqrt{dx+c}\right)-2a\sinh\left(a+b\sqrt{dx+c}\right)}{b^2d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b^2*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2)
)-a*sinh(a+b*(d*x+c)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(48) = 96$.
time = 0.26, size = 110, normalized size = 2.04

$$-\frac{b\left(\frac{\left((dx+c)b^2e^a-2\sqrt{dx+c}\,be^a+2\,e^a\right)e^{\left(\sqrt{dx+c}\,b\right)}}{b^3}+\frac{\left((dx+c)b^2+2\sqrt{dx+c}\,b+2\right)e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^3}\right)-2\,(dx+c)\cosh\left(\sqrt{dx+c}\,b+a\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*e^a + 2*e^a)*e^(sqrt(d*x +
c)*b)/b^3 + ((d*x + c)*b^2 + 2*sqrt(d*x + c)*b + 2)*e^(-sqrt(d*x + c)*b - a
)/b^3) - 2*(d*x + c)*cosh(sqrt(d*x + c)*b + a))/d
```

Fricas [A]
time = 0.43, size = 44, normalized size = 0.81

$$\frac{2\left(\sqrt{dx+c}\,b\sinh\left(\sqrt{dx+c}\,b+a\right)-\cosh\left(\sqrt{dx+c}\,b+a\right)\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*x + c)*b*sinh(sqrt(d*x + c)*b + a) - cosh(sqrt(d*x + c)*b + a))/(
b^2*d)
```

Sympy [A]
time = 0.16, size = 65, normalized size = 1.20

$$\begin{cases} x\cosh(a) & \text{for } b=0\wedge(b=0\vee d=0) \\ x\cosh\left(a+b\sqrt{c}\right) & \text{for } d=0 \\ \frac{2\sqrt{c+dx}\sinh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\cosh\left(a+b\sqrt{c+dx}\right)}{b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*cosh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cosh(a + b*sqrt
(c)), Eq(d, 0)), (2*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b*d) - 2*cosh(
a + b*sqrt(c + d*x))/(b**2*d), True))
```

Giac [A]
time = 0.40, size = 65, normalized size = 1.20

$$\frac{\left(\sqrt{dx+c}\,b-1\right)e^{\left(\sqrt{dx+c}\,b+a\right)}}{b^2d}-\frac{\left(\sqrt{dx+c}\,b+1\right)e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] (sqrt(d*x + c)*b - 1)*e^(sqrt(d*x + c)*b + a)/(b^2*d) - (sqrt(d*x + c)*b +
1)*e^(-sqrt(d*x + c)*b - a)/(b^2*d)
```

Mupad [B]
time = 0.96, size = 43, normalized size = 0.80

$$-\frac{2\left(\cosh\left(a+b\sqrt{c+d\,x}\right)-b\sinh\left(a+b\sqrt{c+d\,x}\right)\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] -(2*(cosh(a + b*(c + d*x)^(1/2)) - b*sinh(a + b*(c + d*x)^(1/2))*(c + d*x)^
(1/2)))/(b^2*d)
```

3.62 $\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx$

Optimal. Leaf size=124

$$\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)+\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)-\sinh\left(a+b\sqrt{c}\right)$$

```
[Out] Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*cosh(a-b*c^(1/2))+Chi(b*(c^(1/2)-(d*x+c)^(1/
2)))*cosh(a+b*c^(1/2))+Shi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))-Shi
(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))
```

Rubi [A]

time = 0.21, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5473, 5401, 3384, 3379, 3382}

$$\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)+\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)-\sinh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)+\sinh\left(a-b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] Cosh[a + b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] - Sqrt[c + d*x])] + Cosh[a - b*
Sqrt[c]]*CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])] - Sinh[a + b*Sqrt[c]]*Si
nhIntegral[b*(Sqrt[c] - Sqrt[c + d*x])] + Sinh[a - b*Sqrt[c]]*SinhIntegral[
b*(Sqrt[c] + Sqrt[c + d*x])]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx &= \text{Subst}\left(\int \frac{\cosh\left(a+b\sqrt{x}\right)}{-c+x}\,dx, x, c+dx\right)\\
&= 2\text{Subst}\left(\int \frac{x\cosh(a+bx)}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= 2\text{Subst}\left(\int \left(-\frac{\cosh(a+bx)}{2\left(\sqrt{c}-x\right)}+\frac{\cosh(a+bx)}{2\left(\sqrt{c}+x\right)}\right)\,dx, x, \sqrt{c+dx}\right)\\
&= -\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)+\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt{c}+x}\,dx, x, \sqrt{\right.\\
&= \cosh\left(a-b\sqrt{c}\right)\text{Subst}\left(\int \frac{\cosh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)-\cosh\left(a+b\right.\\
&= \cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\sqrt{c}-b\right.
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 127, normalized size = 1.02

$$\frac{1}{2}e^{-a-b\sqrt{c}}\left(\text{Ei}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)+e^{2\left(a+b\sqrt{c}\right)}\text{Ei}\left(b\left(-\sqrt{c}+\sqrt{c+dx}\right)\right)+e^{2b\sqrt{c}}\text{Ei}\left(-b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+e^{2a}\text{Ei}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] (E^(-a - b*Sqrt[c])*(ExpIntegralEi[b*(Sqrt[c] - Sqrt[c + d*x])] + E^(2*(a +
 b*Sqrt[c]))*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] + E^(2*b*Sqrt[c])*
ExpIntegralEi[-(b*(Sqrt[c] + Sqrt[c + d*x]))] + E^(2*a)*ExpIntegralEi[b*(Sq
rt[c] + Sqrt[c + d*x])]))/2
```

Maple [F]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+b\sqrt{dx+c}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*(d*x+c)^(1/2))/x,x)
```

```
[Out] int(cosh(a+b*(d*x+c)^(1/2))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh(sqrt(d*x + c)*b + a)/x, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(102) = 204$.
time = 0.35, size = 217, normalized size = 1.75

$$\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}\,b-\sqrt{b^2c}\right)+\mathrm{Ei}\left(-\sqrt{dx+c}\,b+\sqrt{b^2c}\right)\right)\cosh\left(a+\sqrt{b^2c}\right)+\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}\,b+\sqrt{b^2c}\right)+\mathrm{Ei}\left(-\sqrt{dx+c}\,b-\sqrt{b^2c}\right)\right)\cosh\left(-a+\sqrt{b^2c}\right)+\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}\,b-\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+c}\,b+\sqrt{b^2c}\right)\right)\sinh\left(a+\sqrt{b^2c}\right)-\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}\,b+\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+c}\,b-\sqrt{b^2c}\right)\right)\sinh\left(-a+\sqrt{b^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b + sqrt(b^2*c))
)*cosh(a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + Ei(-sqrt
(d*x + c)*b - sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*
b - sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)
) - 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b - sqrt(b^2
*c)))*sinh(-a + sqrt(b^2*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)**(1/2))/x,x)
```

```
[Out] Integral(cosh(a + b*sqrt(c + d*x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x,x, algorithm="giac")
[Out] integrate(cosh(sqrt(d*x + c)*b + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,\sqrt{c+d\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*(c + d*x)^(1/2))/x,x)
[Out] int(cosh(a + b*(c + d*x)^(1/2))/x, x)
```

3.63 $\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$

Optimal. Leaf size=182

$$-\frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}-\frac{bd\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sinh\left(a-b\sqrt{c}\right)}{2\sqrt{c}}+\frac{bd\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\sinh}{2\sqrt{c}}$$

```
[Out] -cosh(a+b*(d*x+c)^(1/2))/x-1/2*b*d*cosh(a+b*c^(1/2))*Shi(b*(c^(1/2)-(d*x+c)
^(1/2)))/c^(1/2)-1/2*b*d*cosh(a-b*c^(1/2))*Shi(b*(c^(1/2)+(d*x+c)^(1/2)))/c
^(1/2)-1/2*b*d*Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))/c^(1/2)+1/2
*b*d*Chi(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))/c^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5473, 5397, 5388, 3384, 3379, 3382}

$$-\frac{bd\sinh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}}+\frac{bd\sinh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\cosh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\cosh\left(a-b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Cosh[a + b*Sqrt[c + d*x]]/x) - (b*d*CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*
x])]*Sinh[a - b*Sqrt[c]])/(2*Sqrt[c]) + (b*d*CoshIntegral[b*(Sqrt[c] - Sqrt
[c + d*x])]*Sinh[a + b*Sqrt[c]])/(2*Sqrt[c]) - (b*d*Cosh[a + b*Sqrt[c]]*Sin
hIntegral[b*(Sqrt[c] - Sqrt[c + d*x])])/(2*Sqrt[c]) - (b*d*Cosh[a - b*Sqrt[
c]]*SinhIntegral[b*(Sqrt[c] + Sqrt[c + d*x])])/(2*Sqrt[c])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh \left(a+b\sqrt{c+dx}\right)}{x^2}\,dx &= d\text{Subst}\left(\int \frac{\cosh \left(a+b\sqrt{x}\right)}{(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\text{Subst}\left(\int \frac{x\cosh(a+bx)}{\left(c-x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\cosh \left(a+b\sqrt{c+dx}\right)}{x} - (bd)\text{Subst}\left(\int \frac{\sinh(a+bx)}{c-x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\cosh \left(a+b\sqrt{c+dx}\right)}{x} - (bd)\text{Subst}\left(\int \left(\frac{\sinh(a+bx)}{2\sqrt{c}\left(\sqrt{c}-x\right)} + \frac{\sinh(a+b}{2\sqrt{c}\left(\sqrt{c}\right.}\right.\right.\\
&= -\frac{\cosh \left(a+b\sqrt{c+dx}\right)}{x} - \frac{(bd)\text{Subst}\left(\int \frac{\sinh(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} - \frac{(bd)}{}\\
&= -\frac{\cosh \left(a+b\sqrt{c+dx}\right)}{x} - \frac{\left(bd\cosh\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx\right.}{2\sqrt{c}}\\
&= -\frac{\cosh \left(a+b\sqrt{c+dx}\right)}{x} - \frac{bd\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sinh\left(a-b\sqrt{c}\right)}{2\sqrt{c}} +
\end{aligned}
$$

Mathematica [A]
time = 1.29, size = 199, normalized size = 1.09

$$
\frac{e^{-a}\left(-2\sqrt{c}\,e^{-b\sqrt{c+dx}} - bde^{-b\sqrt{c}}x\text{Ei}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right) + bde^{b\sqrt{c}}x\text{Ei}\left(-b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\right) + e^{a}\left(-2\sqrt{c}\,e^{b\sqrt{c+dx}} + bde^{b\sqrt{c}}x\text{Ei}\left(b\left(-\sqrt{c}+\sqrt{c+dx}\right)\right) - bde^{-b\sqrt{c}}x\text{Ei}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\right)}{4\sqrt{c}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (((-2*Sqrt[c])/E^(b*Sqrt[c + d*x]) - (b*d*x*ExpIntegralEi[b*(Sqrt[c] - Sqrt
[c + d*x])])/E^(b*Sqrt[c]) + b*d*E^(b*Sqrt[c])*x*ExpIntegralEi[-(b*(Sqrt[c]
 + Sqrt[c + d*x]))])/E^a + E^a*(-2*Sqrt[c]*E^(b*Sqrt[c + d*x]) + b*d*E^(b*S
qrt[c])*x*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] - (b*d*x*ExpIntegralE
i[b*(Sqrt[c] + Sqrt[c + d*x])])/E^(b*Sqrt[c])))/(4*Sqrt[c]*x)
```

Maple [F]
time = 0.60, size = 0, normalized size = 0.00

$$
\int \frac{\cosh \left(a+b\sqrt{dx+c}\right)}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*(d*x+c)^(1/2))/x^2,x)
```

```
[Out] int(cosh(a+b*(d*x+c)^(1/2))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(sqrt(d*x + c)*b + a)/x^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(142) = 284$.
time = 0.47, size = 317, normalized size = 1.74

$$-\frac{4\,c\cosh\left(\sqrt{dx+c}\,b+a\right)-\left(\sqrt{b^2c}\,dx\mathrm{Ei}\left(\sqrt{dx+c}\,b-\sqrt{b^2c}\right)-\sqrt{b^2c}\,dx\mathrm{Ei}\left(-\sqrt{dx+c}\,b+\sqrt{b^2c}\right)\right)\cosh\left(a+\sqrt{b^2c}\right)+\left(\sqrt{b^2c}\,dx\mathrm{Ei}\left(\sqrt{dx+c}\,b+\sqrt{b^2c}\right)-\sqrt{b^2c}\,dx\mathrm{Ei}\left(-\sqrt{dx+c}\,b-\sqrt{b^2c}\right)\right)\cosh\left(-a+\sqrt{b^2c}\right)-\left(\sqrt{b^2c}\,dx\mathrm{Ei}\left(\sqrt{dx+c}\,b-\sqrt{b^2c}\right)+\sqrt{b^2c}\,dx\mathrm{Ei}\left(-\sqrt{dx+c}\,b+\sqrt{b^2c}\right)\right)\sinh\left(a+\sqrt{b^2c}\right)-\left(\sqrt{b^2c}\,dx\mathrm{Ei}\left(\sqrt{dx+c}\,b+\sqrt{b^2c}\right)+\sqrt{b^2c}\,dx\mathrm{Ei}\left(-\sqrt{dx+c}\,b-\sqrt{b^2c}\right)\right)\sinh\left(-a+\sqrt{b^2c}\right)}{4\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*cosh(sqrt(d*x + c)*b + a) - (sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b -
 sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*cosh(a
+ sqrt(b^2*c)) + (sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) - sqrt(
b^2*c)*d*x*Ei(-sqrt(d*x + c)*b - sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) - (sq
rt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + sqrt(b^2*c)*d*x*Ei(-sqrt(
d*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)) - (sqrt(b^2*c)*d*x*Ei(sqrt
(d*x + c)*b + sqrt(b^2*c)) + sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b - sqrt(b^2
*c)))*sinh(-a + sqrt(b^2*c)))/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)**(1/2))/x**2,x)
```

```
[Out] Integral(cosh(a + b*sqrt(c + d*x))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(sqrt(d*x + c)*b + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,\sqrt{c+d\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*(c + d*x)^(1/2))/x^2,x)
```

```
[Out] int(cosh(a + b*(c + d*x)^(1/2))/x^2, x)
```

3.64 $\int x^2 \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=537

$$\frac{720c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3}-\frac{120960\sqrt[3]{c+dx}\,\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3}-\frac{6c^2\sqrt[3]{c+dx}\,\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}+$$

```
[Out] 720*c*cosh(a+b*(d*x+c)^(1/3))/b^6/d^3-120960*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)
^(1/3))/b^8/d^3-6*c^2*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1/3))/b^2/d^3+360*c*(
d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/b^4/d^3-20160*(d*x+c)*cosh(a+b*(d*x+c)
^(1/3))/b^6/d^3+30*c*(d*x+c)^(4/3)*cosh(a+b*(d*x+c)^(1/3))/b^2/d^3-1008*(d*
x+c)^(5/3)*cosh(a+b*(d*x+c)^(1/3))/b^4/d^3-24*(d*x+c)^(7/3)*cosh(a+b*(d*x+c
)^(1/3))/b^2/d^3+120960*sinh(a+b*(d*x+c)^(1/3))/b^9/d^3+6*c^2*sinh(a+b*(d*x
+c)^(1/3))/b^3/d^3-720*c*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3))/b^5/d^3+6048
0*(d*x+c)^(2/3)*sinh(a+b*(d*x+c)^(1/3))/b^7/d^3+3*c^2*(d*x+c)^(2/3)*sinh(a+
b*(d*x+c)^(1/3))/b/d^3-120*c*(d*x+c)*sinh(a+b*(d*x+c)^(1/3))/b^3/d^3+5040*(
d*x+c)^(4/3)*sinh(a+b*(d*x+c)^(1/3))/b^5/d^3-6*c*(d*x+c)^(5/3)*sinh(a+b*(d*
x+c)^(1/3))/b/d^3+168*(d*x+c)^2*sinh(a+b*(d*x+c)^(1/3))/b^3/d^3+3*(d*x+c)^(
8/3)*sinh(a+b*(d*x+c)^(1/3))/b/d^3
```

Rubi [A]
time = 0.49, antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5473, 1607, 5395, 3377, 2717, 2718}

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (720*c*Cosh[a + b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*(c + d*x)^(1/3)*Cos
h[a + b*(c + d*x)^(1/3)])/(b^8*d^3) - (6*c^2*(c + d*x)^(1/3)*Cosh[a + b*(c
+ d*x)^(1/3)])/(b^2*d^3) + (360*c*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3
)])/(b^4*d^3) - (20160*(c + d*x)*Cosh[a + b*(c + d*x)^(1/3)])/(b^6*d^3) + (
30*c*(c + d*x)^(4/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b^2*d^3) - (1008*(c + d*
x)^(5/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b^4*d^3) - (24*(c + d*x)^(7/3)*Cosh[
a + b*(c + d*x)^(1/3)])/(b^2*d^3) + (120960*Sinh[a + b*(c + d*x)^(1/3)])/(b
^9*d^3) + (6*c^2*Sinh[a + b*(c + d*x)^(1/3)])/(b^3*d^3) - (720*c*(c + d*x)^
(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^5*d^3) + (60480*(c + d*x)^(2/3)*Sinh[
a + b*(c + d*x)^(1/3)])/(b^7*d^3) + (3*c^2*(c + d*x)^(2/3)*Sinh[a + b*(c +
d*x)^(1/3)])/(b*d^3) - (120*c*(c + d*x)*Sinh[a + b*(c + d*x)^(1/3)])/(b^3*d
^3) + (5040*(c + d*x)^(4/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^5*d^3) - (6*c*(
c + d*x)^(5/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b*d^3) + (168*(c + d*x)^2*Sinh
[a + b*(c + d*x)^(1/3)])/(b^3*d^3) + (3*(c + d*x)^(8/3)*Sinh[a + b*(c + d*x
)^(1/3)])/(b*d^3)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \cosh\left(a+b\sqrt[3]{c+dx}\right) dx = \frac{\text{Subst}\left(\int (-c+x)^2 \cosh\left(a+b\sqrt[3]{x}\right) dx, x, c+dx\right)}{d^3}$$

$$= \frac{3\text{Subst}\left(\int \left(-cx+x^4\right)^2 \cosh(a+bx)\, dx, x, \sqrt[3]{c+dx}\right)}{d^3}$$

$$= \frac{3\text{Subst}\left(\int x^2\left(-c+x^3\right)^2 \cosh(a+bx)\, dx, x, \sqrt[3]{c+dx}\right)}{d^3}$$

$$= \frac{3\text{Subst}\left(\int \left(c^2x^2\cosh(a+bx) - 2cx^5\cosh(a+bx) + x^8\cosh(a+bx)\right) dx, x\right.}{d^3}$$

$$= \frac{3\text{Subst}\left(\int x^8 \cosh(a+bx)\, dx, x, \sqrt[3]{c+dx}\right)}{d^3} - \frac{(6c)\text{Subst}\left(\int x^5 \cosh(a+bx\right.}{d^3}$$

$$= \frac{3c^2(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right.}{bd^3}$$

$$= -\frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{30c(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+d\right.}{b^2d^3}$$

$$= -\frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{30c(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+d\right.}{b^2d^3}$$

$$= -\frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+\right.}{b^4d^3}$$

$$= -\frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+\right.}{b^4d^3}$$

$$= \frac{720c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c}{}$$

$$= \frac{720c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c}{}$$

$$= \frac{720c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} - \frac{6}{}$$

$$= \frac{720c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} - \frac{6}{}$$

Mathematica [A]
time = 0.52, size = 381, normalized size = 0.71

$$\frac{3\left(40320-40320b\sqrt[3]{c+dx}+20160b^2(c+dx)^{2/3}+b^8d^2x^2(c+dx)^{2/3}-2b^7dx\sqrt[3]{c+dx}(3c+4dx)+240b^4\sqrt[3]{c+dx}(6c+7dx)-24b^5(c+dx)^{2/3}(9c+14dx)-240b^3(27c+28dx)+2b^6(9c^2+36cdx+28d^2x^2)\right)(\cosh(a)+\sinh(a))\left(\cosh\left(b\sqrt[3]{c+dx}\right)+\sinh\left(b\sqrt[3]{c+dx}\right)\right)+\left(40320+40320b\sqrt[3]{c+dx}+20160b^2(c+dx)^{2/3}+b^8d^2x^2(c+dx)^{2/3}+2b^7dx\sqrt[3]{c+dx}(3c+4dx)+240b^4\sqrt[3]{c+dx}(6c+7dx)+24b^5(c+dx)^{2/3}(9c+14dx)+240b^3(27c+28dx)+2b^6(9c^2+36cdx+28d^2x^2)\right)\left(-3\cosh\left(a+b\sqrt[3]{c+dx}\right)+3\sinh\left(a+b\sqrt[3]{c+dx}\right)\right)}{2b^9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*(40320 - 40320*b*(c + d*x)^(1/3) + 20160*b^2*(c + d*x)^(2/3) + b^8*d^2*x
^2*(c + d*x)^(2/3) - 2*b^7*d*x*(c + d*x)^(1/3)*(3*c + 4*d*x) + 240*b^4*(c +
 d*x)^(1/3)*(6*c + 7*d*x) - 24*b^5*(c + d*x)^(2/3)*(9*c + 14*d*x) - 240*b^3
*(27*c + 28*d*x) + 2*b^6*(9*c^2 + 36*c*d*x + 28*d^2*x^2))*(Cosh[a] + Sinh[a
])*(Cosh[b*(c + d*x)^(1/3)] + Sinh[b*(c + d*x)^(1/3)]) + (40320 + 40320*b*(
c + d*x)^(1/3) + 20160*b^2*(c + d*x)^(2/3) + b^8*d^2*x^2*(c + d*x)^(2/3) +
2*b^7*d*x*(c + d*x)^(1/3)*(3*c + 4*d*x) + 240*b^4*(c + d*x)^(1/3)*(6*c + 7*
d*x) + 24*b^5*(c + d*x)^(2/3)*(9*c + 14*d*x) + 240*b^3*(27*c + 28*d*x) + 2*
b^6*(9*c^2 + 36*c*d*x + 28*d^2*x^2))*(-3*Cosh[a + b*(c + d*x)^(1/3)] + 3*Si
nh[a + b*(c + d*x)^(1/3)]))/(2*b^9*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. $2(477) = 954$.
time = 1.68, size = 1815, normalized size = 3.38

method	result	size
derivativedivides	Expression too large to display	1815
default	Expression too large to display	1815

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^3/b^3*(-10/b^3*a^4*c*((a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))-cosh(
a+b*(d*x+c)^(1/3)))+20/b^3*a^3*c*((a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1
/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+2*sinh(a+b*(d*x+c)^(1/3)
))-20/b^3*a^2*c*((a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))-3*(a+b*(d*x+
c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^
(1/3))-6*cosh(a+b*(d*x+c)^(1/3)))+10/b^3*a*c*((a+b*(d*x+c)^(1/3))^4*sinh(a+
b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))+12*(a+b*(d
*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x
+c)^(1/3))+24*sinh(a+b*(d*x+c)^(1/3)))+c^2*a^2*sinh(a+b*(d*x+c)^(1/3))+c^2*
((a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a
+b*(d*x+c)^(1/3))+2*sinh(a+b*(d*x+c)^(1/3)))-2*c^2*a*((a+b*(d*x+c)^(1/3))*s
inh(a+b*(d*x+c)^(1/3))-cosh(a+b*(d*x+c)^(1/3)))+2/b^3*a^5*c*sinh(a+b*(d*x+c
)^(1/3))+70/b^6*a^4*((a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))-4*(a+b*(
d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(
d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+24*sinh(a+b*(d
*x+c)^(1/3)))-56/b^6*a^5*((a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))-3*(
a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))*sinh(a+b
*(d*x+c)^(1/3))-6*cosh(a+b*(d*x+c)^(1/3)))+28/b^6*a^6*((a+b*(d*x+c)^(1/3))^
2*sinh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+2*s
```

```
inh(a+b*(d*x+c)^(1/3)))-2/b^3*c*((a+b*(d*x+c)^(1/3))^5*sinh(a+b*(d*x+c)^(1/
3))-5*(a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^
3*sinh(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))+
120*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))-120*cosh(a+b*(d*x+c)^(1/3))
)-56/b^6*a^3*((a+b*(d*x+c)^(1/3))^5*sinh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^
(1/3))^4*cosh(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^
(1/3))-60*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1
/3))*sinh(a+b*(d*x+c)^(1/3))-120*cosh(a+b*(d*x+c)^(1/3)))+28/b^6*a^2*((a+b*
(d*x+c)^(1/3))^6*sinh(a+b*(d*x+c)^(1/3))-6*(a+b*(d*x+c)^(1/3))^5*cosh(a+b*(
d*x+c)^(1/3))+30*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))-120*(a+b*(d*
x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))+360*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d
*x+c)^(1/3))-720*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+720*sinh(a+b*(
d*x+c)^(1/3)))-8/b^6*a*((a+b*(d*x+c)^(1/3))^7*sinh(a+b*(d*x+c)^(1/3))-7*(a+
b*(d*x+c)^(1/3))^6*cosh(a+b*(d*x+c)^(1/3))+42*(a+b*(d*x+c)^(1/3))^5*sinh(a+
b*(d*x+c)^(1/3))-210*(a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))+840*(a+b
*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))-2520*(a+b*(d*x+c)^(1/3))^2*cosh(a
+b*(d*x+c)^(1/3))+5040*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))-5040*cos
h(a+b*(d*x+c)^(1/3)))-8/b^6*a^7*((a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3)
)-cosh(a+b*(d*x+c)^(1/3)))+1/b^6*a^8*sinh(a+b*(d*x+c)^(1/3))+1/b^6*((a+b*(d
*x+c)^(1/3))^8*sinh(a+b*(d*x+c)^(1/3))-8*(a+b*(d*x+c)^(1/3))^7*cosh(a+b*(d*
x+c)^(1/3))+56*(a+b*(d*x+c)^(1/3))^6*sinh(a+b*(d*x+c)^(1/3))-336*(a+b*(d*x+
c)^(1/3))^5*cosh(a+b*(d*x+c)^(1/3))+1680*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*
x+c)^(1/3))-6720*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))+20160*(a+b*(
d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-40320*(a+b*(d*x+c)^(1/3))*cosh(a+b*
(d*x+c)^(1/3))+40320*sinh(a+b*(d*x+c)^(1/3))))
```

Maxima [A]
time = 0.28, size = 642, normalized size = 1.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*d^3*x^3*cosh((d*x + c)^(1/3)*b + a) + (c^3*e^((d*x + c)^(1/3)*b + a)
/b + c^3*e^(-(d*x + c)^(1/3)*b - a)/b - 3*((d*x + c)*b^3*e^a - 3*(d*x + c)^
(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c^2*e^((d*x + c)^(1/3)*b)/
b^4 - 3*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*c
^2*e^(-(d*x + c)^(1/3)*b - a)/b^4 + 3*((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5
/3)*b^5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x
 + c)^(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*c*e^((d*x + c)^(
1/3)*b)/b^7 + 3*((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/
3)*b^4 + 120*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*
b + 720)*c*e^(-(d*x + c)^(1/3)*b - a)/b^7 - ((d*x + c)^3*b^9*e^a - 9*(d*x +
 c)^(8/3)*b^8*e^a + 72*(d*x + c)^(7/3)*b^7*e^a - 504*(d*x + c)^2*b^6*e^a +
```

```
3024*(d*x + c)^(5/3)*b^5*e^a - 15120*(d*x + c)^(4/3)*b^4*e^a + 60480*(d*x +
 c)*b^3*e^a - 181440*(d*x + c)^(2/3)*b^2*e^a + 362880*(d*x + c)^(1/3)*b*e^a
 - 362880*e^a)*e^((d*x + c)^(1/3)*b)/b^10 - ((d*x + c)^3*b^9 + 9*(d*x + c)^
(8/3)*b^8 + 72*(d*x + c)^(7/3)*b^7 + 504*(d*x + c)^2*b^6 + 3024*(d*x + c)^(
5/3)*b^5 + 15120*(d*x + c)^(4/3)*b^4 + 60480*(d*x + c)*b^3 + 181440*(d*x +
c)^(2/3)*b^2 + 362880*(d*x + c)^(1/3)*b + 362880)*e^(-(d*x + c)^(1/3)*b - a
)/b^10)*b)/d^3
```

Fricas [**A**]
time = 0.35, size = 181, normalized size = 0.34

$$-\frac{3\left(2\left(3360\,b^{3}dx+3240\,b^{3}c+12\,(14\,b^{5}dx+9\,b^{5}c)(dx+c)^{\frac{2}{3}}+(4\,b^{7}d^{2}x^{2}+3\,b^{7}cdx+20160\,b)(dx+c)^{\frac{1}{3}}\right)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(56\,b^{6}d^{2}x^{2}+72\,b^{6}cdx+18\,b^{6}c^{2}+(b^{8}d^{2}x^{2}+20160\,b^{2})(dx+c)^{\frac{2}{3}}+240\,(7\,b^{4}dx+6\,b^{4}c)(dx+c)^{\frac{1}{3}}+40320\right)\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^{9}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*(2*(3360*b^3*d*x + 3240*b^3*c + 12*(14*b^5*d*x + 9*b^5*c)*(d*x + c)^(2/3
) + (4*b^7*d^2*x^2 + 3*b^7*c*d*x + 20160*b)*(d*x + c)^(1/3))*cosh((d*x + c)
^(1/3)*b + a) - (56*b^6*d^2*x^2 + 72*b^6*c*d*x + 18*b^6*c^2 + (b^8*d^2*x^2
+ 20160*b^2)*(d*x + c)^(2/3) + 240*(7*b^4*d*x + 6*b^4*c)*(d*x + c)^(1/3) +
40320)*sinh((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\cosh\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x**2*cosh(a + b*(c + d*x)**(1/3)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2163 vs. $2(477) = 954$.
time = 0.45, size = 2163, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*((((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c
^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*
b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c)^(
1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*b^3*
```

c - 2*((d*x + c)^(1/3)*b + a)*b^6*c^2 + 2*a*b^6*c^2 + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)*a^7 + a^8 + 10*((d*x + c)^(1/3)*b + a)^4*b^3*c - 40*((d*x + c)^(1/3)*b + a)^3*a*b^3*c + 60*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*c - 40*((d*x + c)^(1/3)*b + a)*a^3*b^3*c + 10*a^4*b^3*c + 2*b^6*c^2 - 8*((d*x + c)^(1/3)*b + a)^7 + 56*((d*x + c)^(1/3)*b + a)^6*a - 168*((d*x + c)^(1/3)*b + a)^5*a^2 + 280*((d*x + c)^(1/3)*b + a)^4*a^3 - 280*((d*x + c)^(1/3)*b + a)^3*a^4 + 168*((d*x + c)^(1/3)*b + a)^2*a^5 - 56*((d*x + c)^(1/3)*b + a)*a^6 + 8*a^7 - 40*((d*x + c)^(1/3)*b + a)^3*b^3*c + 120*((d*x + c)^(1/3)*b + a)^2*a*b^3*c - 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c + 40*a^3*b^3*c + 56*((d*x + c)^(1/3)*b + a)^6 - 336*((d*x + c)^(1/3)*b + a)^5*a + 840*((d*x + c)^(1/3)*b + a)^4*a^2 - 1120*((d*x + c)^(1/3)*b + a)^3*a^3 + 840*((d*x + c)^(1/3)*b + a)^2*a^4 - 336*((d*x + c)^(1/3)*b + a)*a^5 + 56*a^6 + 120*((d*x + c)^(1/3)*b + a)^2*b^3*c - 240*((d*x + c)^(1/3)*b + a)*a*b^3*c + 120*a^2*b^3*c - 336*((d*x + c)^(1/3)*b + a)^5 + 1680*((d*x + c)^(1/3)*b + a)^4*a - 3360*((d*x + c)^(1/3)*b + a)^3*a^2 + 3360*((d*x + c)^(1/3)*b + a)^2*a^3 - 1680*((d*x + c)^(1/3)*b + a)*a^4 + 336*a^5 - 240*((d*x + c)^(1/3)*b + a)*b^3*c + 240*a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b + a)^3*a + 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a)*a^3 + 1680*a^4 + 240*b^3*c - 6720*((d*x + c)^(1/3)*b + a)^3 + 20160*((d*x + c)^(1/3)*b + a)^2*a - 20160*((d*x + c)^(1/3)*b + a)*a^2 + 6720*a^3 + 20160*((d*x + c)^(1/3)*b + a)^2 - 40320*((d*x + c)^(1/3)*b + a)*a + 20160*a^2 - 40320*(d*x + c)^(1/3)*b + 40320)*e^((d*x + c)^(1/3)*b + a)/(b^8*d^2) - (((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*b^3*c + 2*((d*x + c)^(1/3)*b + a)*b^6*c^2 - 2*a*b^6*c^2 + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)*a^7 + a^8 - 10*((d*x + c)^(1/3)*b + a)^4*b^3*c + 40*((d*x + c)^(1/3)*b + a)^3*a*b^3*c - 60*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*c + 40*((d*x + c)^(1/3)*b + a)*a^3*b^3*c - 10*a^4*b^3*c + 2*b^6*c^2 + 8*((d*x + c)^(1/3)*b + a)^7 - 56*((d*x + c)^(1/3)*b + a)^6*a + 168*((d*x + c)^(1/3)*b + a)^5*a^2 - 280*((d*x + c)^(1/3)*b + a)^4*a^3 + 280*((d*x + c)^(1/3)*b + a)^3*a^4 - 168*((d*x + c)^(1/3)*b + a)^2*a^5 + 56*((d*x + c)^(1/3)*b + a)*a^6 - 8*a^7 - 40*((d*x + c)^(1/3)*b + a)^3*b^3*c + 120*((d*x + c)^(1/3)*b + a)^2*a*b^3*c - 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c + 40*a^3*b^3*c + 56*((d*x + c)^(1/3)*b + a)^6 - 336*((d*x + c)^(1/3)*b + a)^5*a + 840*((d*x + c)^(1/3)*b + a)^4*a^2 - 1120*((d*x + c)^(1/3)*b + a)^3*a^3 + 840*((d*x + c)^(1/3)*b + a)^2*a^4 - 336*((d*x + c)^(1/3)*b + a)*a^5 + 56*a^6 - 120*((d*x + c)^(1/3)*b + a)^2*b^3*c + 240*((d*x + c)^(1/3)*b + a)*a*b^3*c - 120*a^2*b^3*c + 336*((d*x + c)^

```
(1/3)*b + a)^5 - 1680*((d*x + c)^(1/3)*b + a)^4*a + 3360*((d*x + c)^(1/3)*b
 + a)^3*a^2 - 3360*((d*x + c)^(1/3)*b + a)^2*a^3 + 1680*((d*x + c)^(1/3)*b
+ a)*a^4 - 336*a^5 - 240*((d*x + c)^(1/3)*b + a)*b^3*c + 240*a*b^3*c + 1680
*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b + a)^3*a + 10080*((d*x
 + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a)*a^3 + 1680*a^4 - 24
0*b^3*c + 6720*((d*x + c)^(1/3)*b + a)^3 - 20160*((d*x + c)^(1/3)*b + a)^2*
a + 20160*((d*x + c)^(1/3)*b + a)*a^2 - 6720*a^3 + 20160*((d*x + c)^(1/3)*b
 + a)^2 - 40320*((d*x + c)^(1/3)*b + a)*a + 20160*a^2 + 40320*(d*x + c)^(1/
3)*b + 40320)*e^(-(d*x + c)^(1/3)*b - a)/(b^8*d^2))/(b*d)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \cosh\left(a + b\,(c + d\,x)^{1/3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] int(x^2*cosh(a + b*(c + d*x)^(1/3)), x)
```

3.65 $\int x \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=261

$$-\frac{360\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}+\frac{6c\sqrt[3]{c+dx}\,\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}-\frac{15}{}$$

```
[Out] -360*cosh(a+b*(d*x+c)^(1/3))/b^6/d^2+6*c*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1/
3))/b^2/d^2-180*(d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/b^4/d^2-15*(d*x+c)^(4
/3)*cosh(a+b*(d*x+c)^(1/3))/b^2/d^2-6*c*sinh(a+b*(d*x+c)^(1/3))/b^3/d^2+360
*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3))/b^5/d^2-3*c*(d*x+c)^(2/3)*sinh(a+b*(
d*x+c)^(1/3))/b/d^2+60*(d*x+c)*sinh(a+b*(d*x+c)^(1/3))/b^3/d^2+3*(d*x+c)^(5
/3)*sinh(a+b*(d*x+c)^(1/3))/b/d^2
```

Rubi [A]
time = 0.22, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5473, 5395, 3377, 2717, 2718}

$$-\frac{360\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}+\frac{360\sqrt[3]{c+dx}\,\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2}-\frac{180(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{60(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{6c\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}-\frac{15(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{6c\sqrt[3]{c+dx}\,\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{3(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}-\frac{3c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-360*Cosh[a + b*(c + d*x)^(1/3)])/(b^6*d^2) + (6*c*(c + d*x)^(1/3)*Cosh[a
+ b*(c + d*x)^(1/3)])/(b^2*d^2) - (180*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)
^(1/3)])/(b^4*d^2) - (15*(c + d*x)^(4/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b^2*
d^2) - (6*c*Sinh[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (360*(c + d*x)^(1/3)*S
inh[a + b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*c*(c + d*x)^(2/3)*Sinh[a + b*(c
+ d*x)^(1/3)])/(b*d^2) + (60*(c + d*x)*Sinh[a + b*(c + d*x)^(1/3)])/(b^3*d^
2) + (3*(c + d*x)^(5/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx &= \frac{\text{Subst}\left(\int(-c+x)\cosh\left(a+b\sqrt[3]{x}\right)\,dx,x,c+dx\right)}{d^2} \\
&= \frac{3\text{Subst}\left(\int x^2(-c+x^3)\cosh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2} \\
&= \frac{3\text{Subst}\left(\int\left(-cx^2\cosh(a+bx)+x^5\cosh(a+bx)\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2} \\
&= \frac{3\text{Subst}\left(\int x^5\cosh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}-\frac{(3c)\text{Subst}\left(\int x^2\cosh(a+b
\end{aligned}
$$
$$
\begin{aligned}
&= -\frac{3c(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{3(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right.}{bd^2} \\
&= \frac{6c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{15(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\
&= \frac{6c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{15(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\
&= \frac{6c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right.}{b^4d^2} \\
&= \frac{6c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right.}{b^4d^2} \\
&= -\frac{360\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}+\frac{6c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 118, normalized size = 0.45

$$\frac{-3\left(120+60b^2(c+dx)^{2/3}+b^4\sqrt[3]{c+dx}\,(3c+5dx)\right)\cosh\left(a+b\sqrt[3]{c+dx}\right)+3b\left(120\sqrt[3]{c+dx}+b^4dx(c+dx)^{2/3}+2b^2(9c+10dx)\right)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-3*(120 + 60*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(1/3)*(3*c + 5*d*x))*Cosh
[a + b*(c + d*x)^(1/3)] + 3*b*(120*(c + d*x)^(1/3) + b^4*d*x*(c + d*x)^(2/3
) + 2*b^2*(9*c + 10*d*x))*Sinh[a + b*(c + d*x)^(1/3)])/(b^6*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(231) = 462$.
time = 1.66, size = 659, normalized size = 2.52 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^2/b^3*(-1/b^3*a^5*sinh(a+b*(d*x+c)^(1/3))+5/b^3*a^4*((a+b*(d*x+c)^(1/3)
)*sinh(a+b*(d*x+c)^(1/3))-cosh(a+b*(d*x+c)^(1/3)))-10/b^3*a^3*((a+b*(d*x+c)
^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1
/3))+2*sinh(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*((a+b*(d*x+c)^(1/3))^3*sinh(a+b*
(d*x+c)^(1/3))-3*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+
c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))-6*cosh(a+b*(d*x+c)^(1/3)))-5/b^3*a*((a+b*
(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(
d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x
+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+24*sinh(a+b*(d*x+c)^(1/3)))+1/b^3*((a+b*
(d*x+c)^(1/3))^5*sinh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*cosh(a+b*(
d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x
+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+
c)^(1/3))-120*cosh(a+b*(d*x+c)^(1/3)))-c*a^2*sinh(a+b*(d*x+c)^(1/3))+2*c*a*
((a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))-cosh(a+b*(d*x+c)^(1/3)))-c*((a
+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a+b*
(d*x+c)^(1/3))+2*sinh(a+b*(d*x+c)^(1/3))))
```

Maxima [A]
time = 0.29, size = 369, normalized size = 1.41

$$\frac{2\,d^2x^2\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(\frac{c^2e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{b}+\frac{c^2e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b}-\frac{2\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\,e^a\right)ce^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^4}-\frac{2\left((dx+c)b^3+3\,(dx+c)^{\frac{2}{3}}b^2+6\,(dx+c)^{\frac{1}{3}}b+6\right)ce^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b^4}+\frac{\left((dx+c)^2b^6e^a-6\,(dx+c)^{\frac{5}{3}}b^5e^a+30\,(dx+c)^{\frac{4}{3}}b^4e^a-120\,(dx+c)b^3e^a+360\,(dx+c)^{\frac{2}{3}}b^2e^a-720\,(dx+c)^{\frac{1}{3}}be^a+720\,e^a\right)e^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^7}+\frac{\left((dx+c)^2b^6+6\,(dx+c)^{\frac{5}{3}}b^5+30\,(dx+c)^{\frac{4}{3}}b^4+120\,(dx+c)b^3+360\,(dx+c)^{\frac{2}{3}}b^2+720\,(dx+c)^{\frac{1}{3}}b+720\right)e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b^7}\right)b}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d^2*x^2*cosh((d*x + c)^(1/3)*b + a) - (c^2*e^((d*x + c)^(1/3)*b + a)
/b + c^2*e^(-(d*x + c)^(1/3)*b - a)/b - 2*((d*x + c)*b^3*e^a - 3*(d*x + c)^
```

```
(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c*e^((d*x + c)^(1/3)*b)/b^
4 - 2*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*c*e
^(-(d*x + c)^(1/3)*b - a)/b^4 + ((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5/3)*b^
5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x + c)^
(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*e^((d*x + c)^(1/3)*b)/
b^7 + ((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/3)*b^4 + 1
20*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*b + 720)*e
^(-(d*x + c)^(1/3)*b - a)/b^7)*b)/d^2
```

Fricas [**A**]
time = 0.42, size = 109, normalized size = 0.42

$$-\frac{3\left(\left(60\,(dx+c)^{\frac{2}{3}}b^2+(5\,b^4dx+3\,b^4c)(dx+c)^{\frac{1}{3}}+120\right)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^5dx+20\,b^3dx+18\,b^3c+120\,(dx+c)^{\frac{1}{3}}b\right)\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*((60*(d*x + c)^(2/3)*b^2 + (5*b^4*d*x + 3*b^4*c)*(d*x + c)^(1/3) + 120)*
cosh((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^5*d*x + 20*b^3*d*x + 18*b^
3*c + 120*(d*x + c)^(1/3)*b)*sinh((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cosh\left(a+b\sqrt[3]{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x*cosh(a + b*(c + d*x)**(1/3)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(231) = 462.
time = 0.43, size = 707, normalized size = 2.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3/2*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^3*c
+ a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a - 1
0*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*((d*
x + c)^(1/3)*b + a)*a^4 + a^5 - 2*((d*x + c)^(1/3)*b + a)*b^3*c + 2*a*b^3*c
 + 5*((d*x + c)^(1/3)*b + a)^4 - 20*((d*x + c)^(1/3)*b + a)^3*a + 30*((d*x
```

```
+ c)^(1/3)*b + a)^2*a^2 - 20*((d*x + c)^(1/3)*b + a)*a^3 + 5*a^4 + 2*b^3*c
- 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c)^(1/3)*b + a)^2*a - 60*((d*x
+ c)^(1/3)*b + a)*a^2 + 20*a^3 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*x +
 c)^(1/3)*b + a)*a + 60*a^2 - 120*(d*x + c)^(1/3)*b + 120)*e^((d*x + c)^(1/
3)*b + a)/(b^5*d) - (((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b
 + a)*a*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*
b + a)^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^
2*a^3 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 + 2*((d*x + c)^(1/3)*b + a)*b^3
*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d*x + c)^(1/3)*b + a)^3
*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c)^(1/3)*b + a)*a^3 - 5*
a^4 + 2*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c)^(1/3)*b + a)^2
*a - 60*((d*x + c)^(1/3)*b + a)*a^2 + 20*a^3 - 60*((d*x + c)^(1/3)*b + a)^2
 + 120*((d*x + c)^(1/3)*b + a)*a - 60*a^2 - 120*(d*x + c)^(1/3)*b - 120)*e^
(-(d*x + c)^(1/3)*b - a)/(b^5*d))/(b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \cosh\left(a + b\,(c + d\,x)^{1/3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] int(x*cosh(a + b*(c + d*x)^(1/3)), x)
```

3.66 $\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=85

$$-\frac{6\sqrt[3]{c+dx}\,\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{3(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[Out] -6*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1/3))/b^2/d+6*sinh(a+b*(d*x+c)^(1/3))/b^
3/d+3*(d*x+c)^(2/3)*sinh(a+b*(d*x+c)^(1/3))/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5419, 5413, 3377, 2717}

$$\frac{6\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{6\sqrt[3]{c+dx}\,\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{3(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-6*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b^2*d) + (6*Sinh[a + b*(c
 + d*x)^(1/3)])/(b^3*d) + (3*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/(
b*d)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5413

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Cosh[c + d*x^(k*
n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && I
ntegerQ[p]
```

Rule 5419

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Cosh[c + d*x^n])^p, x], x, u], x]
```

```
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \cosh\left(a+b\sqrt[3]{c+dx}\right)\,dx &= \frac{\text{Subst}\left(\int \cosh\left(a+b\sqrt[3]{x}\right)\,dx, x, c+dx\right)}{d}\\
&= \frac{3\text{Subst}\left(\int x^2\cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= \frac{3(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \sqrt[3]{c+d
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{6\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{3(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \\
&= -\frac{6\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{6\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} + \frac{3(c+dx)^{2/}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 65, normalized size = 0.76

$$\frac{-6b\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)+3\left(2+b^2(c+dx)^{2/3}\right)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*(c + d*x)^(1/3)],x]
[Out] (-6*b*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)] + 3*(2 + b^2*(c + d*x)^(2
/3))*Sinh[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

Maple [A]
time = 1.34, size = 133, normalized size = 1.56

method	result
derivativedivides	$\frac{3a^2\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)+3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\sinh\left(a+b(\right.}{b^3d}$
default	$\frac{3a^2\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)-\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)+3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\sinh\left(a+b(\right.}{b^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d/b^3*(a^2*sinh(a+b*(d*x+c)^(1/3))-2*a*((a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x
+c)^(1/3))-cosh(a+b*(d*x+c)^(1/3)))+(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^
(1/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+2*sinh(a+b*(d*x+c)^(1/
3)))
```

Maxima [A]
time = 0.27, size = 136, normalized size = 1.60

$$-\frac{b\left(\frac{\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\,e^a\right)e^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^4}+\frac{\left((dx+c)b^3+3\,(dx+c)^{\frac{2}{3}}b^2+6\,(dx+c)^{\frac{1}{3}}b+6\right)e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b^4}\right)-2\,(dx+c)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(((d*x + c)*b^3*e^a - 3*(d*x + c)^(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)
*b*e^a - 6*e^a)*e^((d*x + c)^(1/3)*b)/b^4 + ((d*x + c)*b^3 + 3*(d*x + c)^(2
/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*e^(-(d*x + c)^(1/3)*b - a)/b^4) - 2*(d*x
 + c)*cosh((d*x + c)^(1/3)*b + a))/d
```

Fricas [A]
time = 0.36, size = 58, normalized size = 0.68

$$-\frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^2+2\right)\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*(2*(d*x + c)^(1/3)*b*cosh((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2
+ 2)*sinh((d*x + c)^(1/3)*b + a))/(b^3*d)
```

Sympy [A]
time = 0.25, size = 94, normalized size = 1.11

$$\begin{cases} x\cosh\left(a\right) & \text{for}\: b=0\wedge\left(b=0\vee d=0\right)\\ x\cosh\left(a+b\sqrt[3]{c}\right) & \text{for}\: d=0\\ \frac{3(c+dx)^{\frac{2}{3}}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd}-\frac{6\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*cosh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cosh(a + b*c**(
1/3)), Eq(d, 0)), (3*(c + d*x)**(2/3)*sinh(a + b*(c + d*x)**(1/3))/(b*d) -
```

```
6*(c + d*x)**(1/3)*cosh(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*sinh(a + b*(c
+ d*x)**(1/3))/(b**3*d), True))
```

Giac [A]
time = 0.41, size = 128, normalized size = 1.51

$$\frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^{2}-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^{2}-2(dx+c)^{\frac{1}{3}}b+2\right)e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{2b^{3}d}-\frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^{2}-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^{2}+2(dx+c)^{\frac{1}{3}}b+2\right)e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{2b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*(((d*x + c)^(1/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2*(d*x
 + c)^(1/3)*b + 2)*e^((d*x + c)^(1/3)*b + a)/(b^3*d) - 3/2*(((d*x + c)^(1/3
)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 + 2*(d*x + c)^(1/3)*b + 2)*e
^(-(d*x + c)^(1/3)*b - a)/(b^3*d)
```

Mupad [B]
time = 0.98, size = 75, normalized size = 0.88

$$\frac{6\sinh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{b^3\,d}-\frac{6\cosh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{1/3}}{b^2\,d}+\frac{3\sinh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{2/3}}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] (6*sinh(a + b*(c + d*x)^(1/3)))/(b^3*d) - (6*cosh(a + b*(c + d*x)^(1/3))*(c
 + d*x)^(1/3))/(b^2*d) + (3*sinh(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3))/(b
*d)
```

3.67 $\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$

Optimal. Leaf size=232

$$\cosh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+$$

```
[Out] Chi(b*(c^(1/3)-(d*x+c)^(1/3)))*cosh(a+b*c^(1/3))+Chi(b*((-1)^(1/3)*c^(1/3)+
(d*x+c)^(1/3)))*cosh(a-(-1)^(1/3)*b*c^(1/3))+Chi(-b*((-1)^(2/3)*c^(1/3)-(d*
x+c)^(1/3)))*cosh(a+(-1)^(2/3)*b*c^(1/3))-Shi(b*(c^(1/3)-(d*x+c)^(1/3)))*si
nh(a+b*c^(1/3))+Shi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*sinh(a-(-1)^(1/3)
*b*c^(1/3))-Shi(b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*sinh(a+(-1)^(2/3)*b*c
^(1/3))
```

Rubi [A]
time = 0.37, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5473, 5401, 3384, 3379, 3382}

$$\cosh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+\cosh\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{-1}\,\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)-\sinh\left(a+b\sqrt[3]{c}\right)\text{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)-\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+\sinh\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\text{Shi}\left(b\left(\sqrt[3]{-1}\,\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] Cosh[a + b*c^(1/3)]*CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))] + Cosh[a +
(-1)^(2/3)*b*c^(1/3)]*CoshIntegral[-(b*((-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3
)))] + Cosh[a - (-1)^(1/3)*b*c^(1/3)]*CoshIntegral[b*((-1)^(1/3)*c^(1/3) +
(c + d*x)^(1/3))] - Sinh[a + b*c^(1/3)]*SinhIntegral[b*(c^(1/3) - (c + d*x)
^(1/3))] - Sinh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3
) - (c + d*x)^(1/3))] + Sinh[a - (-1)^(1/3)*b*c^(1/3)]*SinhIntegral[b*((-1)
^(1/3)*c^(1/3) + (c + d*x)^(1/3))]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx &= \text{Subst}\left(\int \frac{\cosh\left(a+b\sqrt[3]{x}\right)}{-c+x}\,dx, x, c+dx\right)\\
&= 3\text{Subst}\left(\int \frac{x^2\cosh(a+bx)}{-c+x^3}\,dx, x, \sqrt[3]{c+dx}\right)\\
&= 3\text{Subst}\left(\int \left(-\frac{\cosh(a+bx)}{3\left(\sqrt[3]{c}-x\right)}-\frac{\cosh(a+bx)}{3\left(-\sqrt[3]{-1}\sqrt[3]{c}-x\right)}-\frac{\cosh(a+bx)}{3\left((-1)^{2/3}\sqrt[3]{c}-x\right)}\right)\right.\\
&= -\text{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)-\text{Subst}\left(\int \frac{\cosh(a+bx)}{-\sqrt[3]{-1}\sqrt[3]{c}-x}\,dx,\right.\\
&= -\left(\cosh\left(a+b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{\cosh\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)\right)-\cosh\left(\right.\\
&= \cosh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)+\cosh\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\text{Chi}\left(\sqrt[3]{-1}\right.
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.06, size = 231, normalized size = 1.00

$$\frac{1}{2}\left(\text{RootSum}\left[c-\#1^3\&,\cosh(a+b\#1)\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)-\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)\sinh(a+b\#1)-\cosh(a+b\#1)\text{Shi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\sinh(a+b\#1)\text{Shi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)\&\right]+\text{RootSum}\left[c-\#1^3\&,\cosh(a+b\#1)\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)\sinh(a+b\#1)+\cosh(a+b\#1)\text{Shi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\sinh(a+b\#1)\text{Shi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] (RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[b*((c + d*x)^(1/3) - #1)]
 - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a + b*#1] - Cosh[a + b*#1]*S
inhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b*#1]*SinhIntegral[b*((c +
 d*x)^(1/3) - #1)] & ] + RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[b
*((c + d*x)^(1/3) - #1)] + CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a +
b*#1] + Cosh[a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b*
#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] & ])/2
```

Maple [F]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*(d*x+c)^(1/3))/x,x)
```

```
[Out] int(cosh(a+b*(d*x+c)^(1/3))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh((d*x + c)^(1/3)*b + a)/x, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(182) = 364$.
time = 0.45, size = 503, normalized size = 2.17

$$\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b-\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\cosh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)-a\right)+\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b-\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\cosh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+a\right)+\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right)\cosh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+a\right)+\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right)\cosh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)-a\right)+\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b+\left(b^3c\right)^{\frac{1}{3}}\right)\cosh\left(a+\left(b^3c\right)^{\frac{1}{3}}\right)+\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b+\left(-b^3c\right)^{\frac{1}{3}}\right)\cosh\left(-a+\left(-b^3c\right)^{\frac{1}{3}}\right)+\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b-\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\sinh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)-a\right)+\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b-\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\sinh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)+a\right)-\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right)\sinh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)+a\right)-\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b+\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\right)\sinh\left(\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)-a\right)-\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b+\left(b^3c\right)^{\frac{1}{3}}\right)\sinh\left(a+\left(b^3c\right)^{\frac{1}{3}}\right)-\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b+\left(-b^3c\right)^{\frac{1}{3}}\right)\sinh\left(-a+\left(-b^3c\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x,x, algorithm="fricas")
```

```
[Out] 1/2*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(b^3
*c)^(1/3)*(sqrt(-3) + 1) - a) + 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/
3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) + 1/2*Ei(-(d
*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1/3)*
```

```
(sqrt(-3) - 1) + a) + 1/2*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) + 1/2*Ei(-(d*x + c)^(1
/3)*b + (b^3*c)^(1/3))*cosh(a + (b^3*c)^(1/3)) + 1/2*Ei((d*x + c)^(1/3)*b +
 (-b^3*c)^(1/3))*cosh(-a + (-b^3*c)^(1/3)) + 1/2*Ei(-(d*x + c)^(1/3)*b - 1/
2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a)
+ 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-
b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - 1/2*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^
(1/3)*(sqrt(-3) - 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) - 1/2*Ei((
d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-b^3*c)^(1/
3)*(sqrt(-3) - 1) - a) - 1/2*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*sinh(a
+ (b^3*c)^(1/3)) - 1/2*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-b
^3*c)^(1/3))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)**(1/3))/x,x)
```

```
[Out] Integral(cosh(a + b*(c + d*x)**(1/3))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x,x, algorithm="giac")
```

```
[Out] integrate(cosh((d*x + c)^(1/3)*b + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*(c + d*x)^(1/3))/x,x)
```

```
[Out] int(cosh(a + b*(c + d*x)^(1/3))/x, x)
```

3.68 $\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$

Optimal. Leaf size=329

$$-\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}+\frac{bd\,\mathrm{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\sinh\left(a+b\sqrt[3]{c}\right)}{3c^{2/3}}-\frac{\sqrt[3]{-1}\,bd\,\mathrm{Chi}\left(b\left(\sqrt[3]{-1}\,\sqrt[3]{c}+\sqrt[3]{c\cdot}\right.\right.}{3c^{2/}}$$

```
[Out] -cosh(a+b*(d*x+c)^(1/3))/x-1/3*b*d*cosh(a+b*c^(1/3))*Shi(b*(c^(1/3)-(d*x+c)
^(1/3)))/c^(2/3)-1/3*(-1)^(2/3)*b*d*cosh(a+(-1)^(2/3)*b*c^(1/3))*Shi(b*((-1
)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))/c^(2/3)-1/3*(-1)^(1/3)*b*d*cosh(a-(-1)^(1/3
)*b*c^(1/3))*Shi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))/c^(2/3)+1/3*b*d*Chi(
b*(c^(1/3)-(d*x+c)^(1/3)))*sinh(a+b*c^(1/3))/c^(2/3)-1/3*(-1)^(1/3)*b*d*Chi
(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*sinh(a-(-1)^(1/3)*b*c^(1/3))/c^(2/3)
+1/3*(-1)^(2/3)*b*d*Chi(-b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*sinh(a+(-1)^
(2/3)*b*c^(1/3))/c^(2/3)
```

Rubi [A]

time = 0.52, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5473, 5397, 5388, 3384, 3379, 3382}

$$\frac{bd\sinh\left(a+b\sqrt[3]{c}\right)\mathrm{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}-\frac{\sqrt[3]{-1}\,bd\sinh\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\mathrm{Chi}\left(b\left(\sqrt[3]{-1}\,\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}+\frac{(-1)^{2/3}bd\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\mathrm{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}-\frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\mathrm{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}-\frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\mathrm{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}-\frac{\sqrt[3]{-1}\,bd\cosh\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\mathrm{Shi}\left(b\left(\sqrt[3]{-1}\,\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}-\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] -(Cosh[a + b*(c + d*x)^(1/3)]/x) + (b*d*CoshIntegral[b*(c^(1/3) - (c + d*x)
^(1/3))]*Sinh[a + b*c^(1/3)])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*CoshIntegral[b*
((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))]*Sinh[a - (-1)^(1/3)*b*c^(1/3)])/(3*
c^(2/3)) + ((-1)^(2/3)*b*d*CoshIntegral[-(b*((-1)^(2/3)*c^(1/3) - (c + d*x)
^(1/3)))]*Sinh[a + (-1)^(2/3)*b*c^(1/3)])/(3*c^(2/3)) - (b*d*Cosh[a + b*c^(
1/3)]*SinhIntegral[b*(c^(1/3) - (c + d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(2/3
)*b*d*Cosh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3) - (
c + d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Cosh[a - (-1)^(1/3)*b*c^(1/
3)]*SinhIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))])/(3*c^(2/3))
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx &= d\text{Subst}\left(\int \frac{\cosh\left(a+b\sqrt[3]{x}\right)}{(-c+x)^2}\,dx, x, c+dx\right)\\
&= (3d)\text{Subst}\left(\int \frac{x^2\cosh(a+bx)}{(c-x^3)^2}\,dx, x, \sqrt[3]{c+dx}\right)\\
&= -\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x} - (bd)\text{Subst}\left(\int \frac{\sinh(a+bx)}{c-x^3}\,dx, x, \sqrt[3]{c+dx}\right)\\
&= -\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x} - (bd)\text{Subst}\left(\int \left(\frac{\sinh(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}-x\right)} + \frac{\sinh(a-}{3c^{2/3}\left(\sqrt[3]{c}+\right.}\right.\right.\\
&= -\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{(bd)\text{Subst}\left(\int \frac{\sinh(a+bx)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{(bd)}{}\\
&= -\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x} + \frac{\left(bd\cosh\left(a+b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx\right.}{3c^{2/3}}\\
&= -\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x} + \frac{bd\text{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sinh\left(a+b\sqrt[3]{c}\right)}{3c^{2/3}} -
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.36, size = 211, normalized size = 0.64

$$
\frac{bdx\text{RootSum}\left[c-\#1^3\&, \frac{e^{a+b\#1}\text{Ei}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right] + e^{-a}\left(-3e^{-b\sqrt[3]{c+dx}}\left(1+e^{2\left(a+b\sqrt[3]{c+dx}\right)}\right) - bdx\text{RootSum}\left[c-\#1^3\&, \frac{\cosh(b\#1)\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)-\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)\sinh(b\#1)-\cosh(b\#1)\text{Shi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\sinh(b\#1)\text{Shi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right]\right)}{6x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] (b*d*x*RootSum[c - #1^3 & , (E^(a + b*#1)*ExpIntegralEi[b*((c + d*x)^(1/3)
- #1)])/#1^2 & ] + ((-3*(1 + E^(2*(a + b*(c + d*x)^(1/3)))))/E^(b*(c + d*x)
^(1/3)) - b*d*x*RootSum[c - #1^3 & , (Cosh[b*#1]*CoshIntegral[b*((c + d*x)^
(1/3) - #1)] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[b*#1] - Cosh[b*#
1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[b*#1]*SinhIntegral[b*((c +
 d*x)^(1/3) - #1)])/#1^2 & ])/E^a)/(6*x)
```

Maple [F]
time = 0.65, size = 0, normalized size = 0.00

$$
\int \frac{\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*(d*x+c)^(1/3))/x^2,x)
```

```
[Out] int(cosh(a+b*(d*x+c)^(1/3))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh((d*x + c)^(1/3)*b + a)/x^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(245) = 490$.
time = 0.41, size = 706, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*cosh(a +
(b^3*c)^(1/3)) + 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3)
)*cosh(-a + (-b^3*c)^(1/3)) - 2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (
b^3*c)^(1/3))*sinh(a + (b^3*c)^(1/3)) - 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(
1/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-b^3*c)^(1/3)) - (b^3*c)^(1/3)*(sqrt(-3
)*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh
(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) - (-b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x
)*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*
c)^(1/3)*(sqrt(-3) + 1) + a) + (b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei(-(d*x
+ c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1/3)*(sq
rt(-3) - 1) + a) + (-b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei((d*x + c)^(1/3)*b
 + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1
) - a) - (b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^
3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) - (-b
^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*
(sqrt(-3) + 1))*sinh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - (b^3*c)^(1/3)
*(sqrt(-3)*d*x - d*x)*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) -
 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) - (-b^3*c)^(1/3)*(sqrt(-3)*
d*x - d*x)*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1
/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) + 12*c*cosh((d*x + c)^(1/3)*b + a))/(
c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)**(1/3))/x**2,x)
```

```
[Out] Integral(cosh(a + b*(c + d*x)**(1/3))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh((d*x + c)^(1/3)*b + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*(c + d*x)^(1/3))/x^2,x)
```

```
[Out] int(cosh(a + b*(c + d*x)^(1/3))/x^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```